Iraqi Journal of Statistical Sciences

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Bayesian Estimation of TLR distribution under Sq and E linex loss

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Article information

Article history:
Received : March 11 ,2024
Revised: August 27,2024
Accepted: September 7, 2024
Available online:December 1,2024

Keywords:
Topp – Leone family
Rayleigh distribution
Informative prior distributon
linex loss function
quadratic loss function
Expected linex loss function

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Abstract

Is this paper we study statistical properties of TLR .distribution. Bayesian technique is used to estimate the parameters of distribution under Sq and Elinex loss functions. The shape parameter of linex last function treated as random variable. And distributed b~N(μ, 1) The results applied in simulated data with(different sample sizes and parameters) and real data sets and different values of μ.The Elinex is the best estimator when the location parameter is negative except in the case of scale parameter equal (θ) 2.5 The estimators under sq loss function is the best when(μ >0) except θ =2.5 and the sample size n =25 .In real data set the Elinex estimator is the best when (μ=-2) .this result of real data is supportive to the results of simulated data

DOI: 10.33899/iqjoss.2024.185241 , ©Authors, 2024, College of Computer Science and Mathematics, University of Mosul.

1. Introduction

Probability distributions have received agreat attention in order to achieve flexibility in modeling the studied phenomena.Well- known probability distributions lacks flexibility in modeling convoluted data for complex phenomena, which has made the increasing interest of many researchers to generalize known probability distributions to become more flexible in application than the known distributions.

On this basis, various generalizations were made for the distributions, including those based on the Beta distribution or simpler probability distributions than a Beta distribution and their random variables defined within the interval (0,1). Eugene et al (2002) presented Beta distribution family using the cumulative distribution function (c.d.f) of a random variable w~ Beta(a,b) which is defined as :

$$F(x) = \frac{1}{B(a,b)} \int_0^x w^{a-1} \quad (1-w)^{b-1} \quad dw \qquad (1)$$

if (x) is replaced by the cumulative function G (x) of any basic distribution, the family of the generalized

$$F(x) = \frac{1}{B(a,b)} \int_0^{G(x)} w^{a-1} (1-w)^{b-1} \, dw \qquad (2)$$

where is the B(a,b) is the complete Beta function.

Famoy et al (2005) used this generalization on many basic distributions, such as the Weibull distribution and Hanook et al (2013) generalized inverse Weibull distribution using the beta generalization.

Suggestions for probability distributions are simpler than the Beta distribution are defined within closed interval [0,1] such as the kumaraswamy distribution and the Topp-Leone distribution. Cordiero and Castro (2011) introduced the kumaraswamy distributions family as the cumulative function of this family which takes the following form

$F(x) = 1 - (1 - (G(x))^{\alpha})^{\beta}$

Nadarajah and Kotz (2013) introduced Topp-Leone distribution, which is one of the simplest probability distributions that statisticians used as a alternative to the Beta distribution, as its cumulative function takes the following form

$F_{TL(x)} = x^{\alpha}\ (2-x)^{\alpha}$ Where $0 < x < 1,\ \alpha > 0$ (3)

Shomrani et al (2016) used the c.d.f. defined in eq. (3) above to present a new generalization as an alternative for the Beta generalization called the Topp - Leone family of distributions, as the cumulative function for this generalization takes the following form

$F_{TL-G(x)} = \left(G(x)\right)^{\alpha}(2-G(x))^{\alpha}$ $-\infty < x < \infty,\ \alpha > 0$ (4)

Shomrani et al (2016) gave an example of using the generalization defined in equation (4) on the exponential distribution. Al - Saiary and Bakoban (2020) used this generalization on the generalized inverted exponential distribution of the baseline distribution and the distribution properties and estimations were studied according to maximum likelihood.

Rayleigh distribution is one of the probability distributions that belong's to the distributions of survival times and has wide applications in survival analysis, reliability theory, communications and engineering. This distribution has a linear increasing hazard function with time. Therefore it was also used in modeling the survival times of patients with incurable diseases whose degree of risk increases over time. to avoid restricting the use of this distribution various generalization have been made on it on this basis. In this research, the Topp-Leone generalization was used on this distribution and its statistical properties with estimation of its parameters in Bayesian technique under expected linex (E.linex) and Square quadratic loss functions were studied.

The research is divided into six parts, the first part dealt with the introduction, the second part dealt with the Topp - Leone Rayleigh distribution, the third part discusses the properties of the distribution, and the fourth part includes estimating the parameters of the distribution in the Bayesian style, and the fifth part dealt with the practical side, as the theoretical results were applied to the data of the time when an injury which occurred to dialysis patients by months.

I. TOPP-LEONE RAYLEIGH DISTRIBUTION

If the basic distribution of the random variable x is a Rayleigh distribution with the scale parameter θ where the c.d.f. and the p.d.f. are as follows:

$G(x) = 1 - e^{-\theta x^2}$ $x>0,\ \theta>0$ (5)

$g(x) = 2\theta x e^{-\theta x^2}$ $x, \theta > 0$ (6)

Using eq. (no. 5 , 6) in eq.(no.4), we get the c.d.f. of the Topp-Leone Rayleigh Distribution (TLR) as follows:

$F(x) = (1 - e^{-2\theta x^2})^{\alpha}$ $x, \alpha, \theta > 0$ (7)

By taking the first derivative of both sides of the eq. (no.7) with respect to x, the p.d.f. of x is

$$f(x) = \begin{cases} 4\alpha\theta x e^{-2\theta x^2}(1 - e^{-2\theta x^2})^{\alpha-1} & \alpha, \theta, x > 0 \\ 0 & 0.w \end{cases} \quad (8)$$

This distribution is symbolized as an acronym x ~ T LR (α ,θ)

The survival and hazard functions of the TLR distribution are

$$s(x) = 1 - (1 - e^{-2\theta x^2})^{\alpha} \quad (9)$$

$$h(x) = \frac{4\alpha\theta x e^{-2\theta x^2}(1 - e^{-2\theta x^2})^{\alpha-1}}{1 - (1 - e^{-2\theta x^2})^{\alpha}} \quad (10)$$

f(x),F(x), s(x), h(x) have been plotted when
α=(1.5,2.5) and θ =(0.5,1,1.5,2.5)

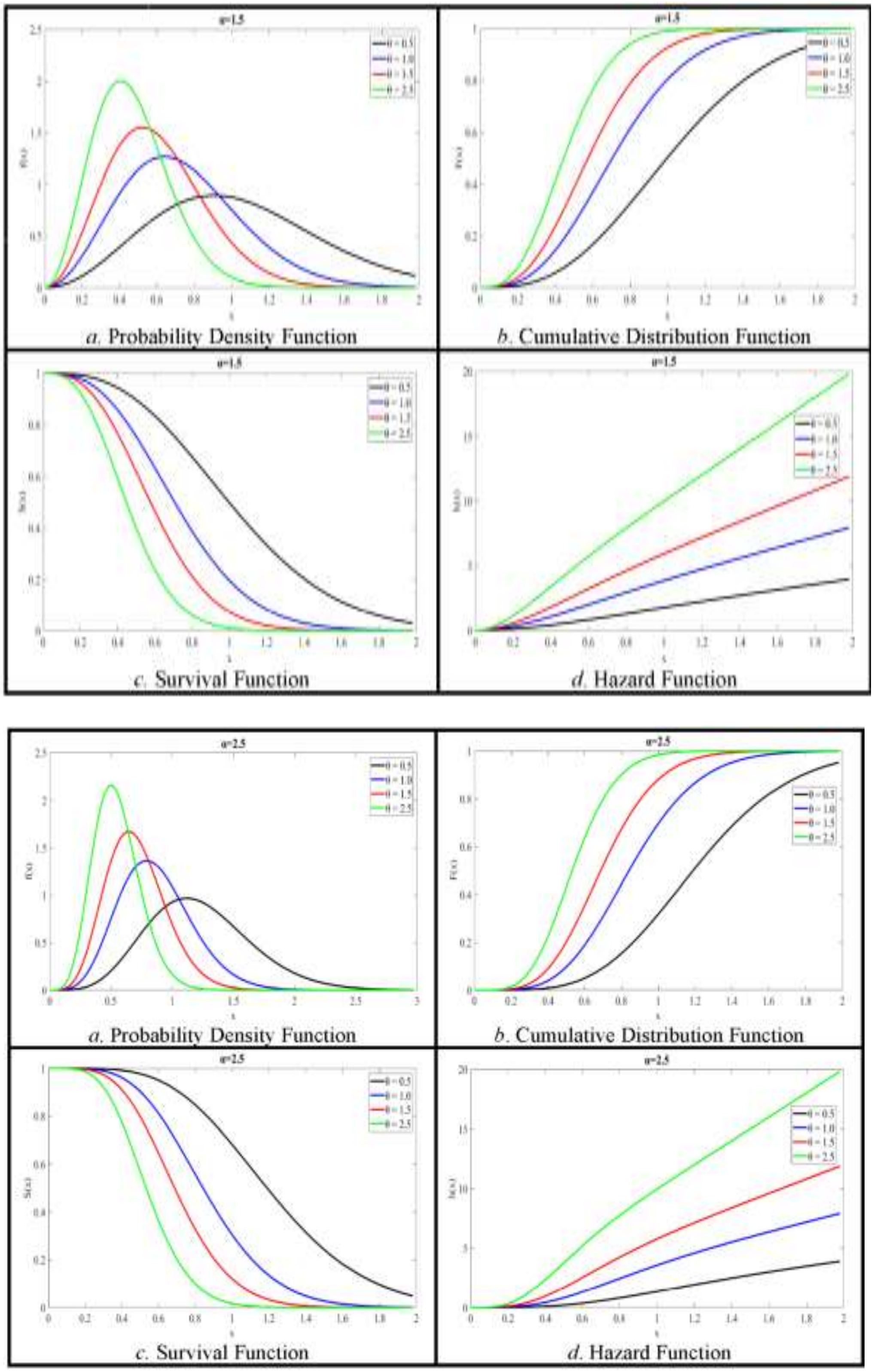

Fig. 1. Plotting f (x),F(x), s (x), h (x) of the TLR distribution at different values of its parameters.

We notice from Figure (1) above that the skewed distribution is positive, and the degree of skewness increases with the increase in the value of the shape parameter (α). As for the hazard function, it takes the (J) shape and its intensity increases.

II. PROPERTIES OF DISTRIBUTION

In this section, some properties of the distribution will be presented which are moments, mode, median, skewness, kurtosis , and quantile function as follows

- **r-order moment about zero**

The r- th moment about zero for the random variable x is

$$Ex^r = 4\alpha\theta \int_0^\infty x^{r+1}\ e^{-2\theta x^2}\ (1-e^{-2\theta x^2})^{(\alpha-1)}\ dx \qquad (11)$$

Using the binomial expansion over the expression $(1-e^{-2\theta x^2})^{\alpha-1}$ and making some mathematical simplifications, we get

$$Ex^r = \sum_{j=0}^{\infty}(-1)^j C_j^{\alpha-1} \frac{\Gamma\frac{r+2}{2}}{((j+1)\theta)^{\frac{r+2}{2}}\ 2^{\frac{r+4}{2}}} \qquad (12)$$

- **Mode**

The mode is the solution of the derivative eq. (no.8) above with respect to x

$$lng(x) = ln4 + ln\alpha + ln\theta + lnx - 2\theta x^2 + (\alpha-1)ln\left(1-e^{-2\theta x^2}\right)$$

$$\frac{\partial lng(x)}{\partial x} = \frac{1}{x} - 4\theta x + \frac{(\alpha-1)4\theta x e^{-2\theta X^2}}{\left(1-e^{-2\theta X^2}\right)}$$

$$\frac{1}{x} - 4\theta x + \frac{4\theta x(\alpha-1)e^{-2\theta x^2}}{1-e^{-2\theta x^2}} = 0 \qquad (13)$$

- **The median and the quantile function**

The median and the quantile function are found by solving the equation $F(x) = \frac{1}{2}$ and the inverse function F(x) respectively to get $me = \sqrt{\frac{-Ln(1-(\frac{1}{2})^{\frac{1}{\alpha}})}{2\theta}}$

$$xq = \sqrt{\frac{-Ln(1-(q)^{\frac{1}{\alpha}})}{2\theta}} \quad \text{where } 0<q<1 \qquad (14)$$

- **Moment Generating Function**

The moment generating function of x is:

$$\mu x(t) = Ee^{tx} = \int_0^\infty e^{tx} f(x)\, dx$$

$$e^{tx} = \sum_{r=0}^{\infty} \frac{t^r x^r}{r!}$$

$$\mu_x(t) = 4\alpha\theta \sum_{j=0}^{\infty} \sum_{r=0}^{\infty} \frac{t^r}{r!} C_j^{\alpha-1}(-1)^j \frac{\Gamma\frac{r+2}{2}}{((j+1)\theta)^{\frac{r+2}{2}}\ 2^{\frac{r+4}{2}}} \qquad (15)$$

- **Kurtosis and skewness**

The skewness and Kurtosis are found mathematically using the moments about zero by solving the following formulas, respectively (sarhan and zaindin . 2000).

$$\text{Skewness} = \frac{\mu_3 - 3\mu_1\mu_2 + 2{\mu_1}^3}{(\mu_2 - {\mu_1}^2)^{\frac{3}{2}}} \qquad (16)$$

$$\text{Kurtosis} = \frac{\mu_4 - 4\mu_1\mu_3 + 6{\mu_1}^2\mu_2 - 3{\mu_1}^4}{(\mu_2 - {\mu_1}^2)^2} \qquad (17)$$

III.BAYESIAN ESTIMATION

In this section, the two parameters $\alpha\ and\ \theta$ of the TLR distribution are estimated when the two parameters, using the in formative prior, and assuming that these parameters are independent, then the common initial distribution is

$$p(\alpha,\theta) = p(\alpha)p(\theta)$$

We choose the prior distribution for α and θ, which are

α~Gamma($c_{0,}d_0$) and θ~$Gamma\left(a_{0,}b_0\right)$

By providing n of the sample observations, the likelihood function is as follows

$$L(\alpha,\theta) \propto \theta^{n}\alpha^{n}e^{-2\theta\sum xi^{2}}\,\pi_{i=1}^{n}\,(1-e^{-2\theta xi^{2}})^{(\alpha-1)} \tag{18}$$

And joint posterior distribution for α and θ is

$$p(\alpha) \propto \alpha^{c_0-1}e^{-d_0\alpha}$$

$$p(\theta) \propto \theta^{a_0-1}e^{-b_0\theta}$$

$$p(\underline{\delta} \setminus x) \propto L(\alpha,\theta)\;p(\alpha,\theta)$$

$$\propto \theta^{a0+n-1}\;\alpha^{c0+n-1}\;e^{-(2\sum xi^{2}+b0)\theta}\;\;\pi_{i=1}^{n}\,(1-e^{-2\theta xi^{2}})^{(\alpha-1)}\;\;e^{-d0\alpha} \tag{19}$$

Where as $\underline{\delta} = (\alpha\,,\theta)'$

We notice from the common posterior distribution defined in eq. (no.19) above that it is difficult to find the posterior marginal distribution for each parameter. Therefore, Laplace approximation was used. Azevedo, F.A.et al (1994)

The approximate posterior – distribution of $\underline{\delta}$ is as follows

$$p(\underline{\delta} \setminus x) \approx e^{-\frac{(\underline{\delta}-\hat{\underline{\delta}})' w(\underline{\delta}-\hat{\underline{\delta}})}{2}} \tag{20}$$

It is the kernel of N2($\hat{\underline{\delta}}, w$) where $\hat{\underline{\delta}}$ represents ($\hat{\alpha}_{mle,}\hat{\theta}_{mle}$) the posterior mode and W=-[$\frac{\partial^{2} q(\underline{\delta}/x)}{\partial\underline{\delta}\partial\underline{\delta}'}$]

Under the Sq loss function, the approximate Bayes estimator of the parameter vector δ is $\delta\hat{}$ is and the quadratic risk function is tr (w^{-1})

As for estimating the parameters under the linex loss function, we need to find the approximate posterior marginal distribution for each parameter and describe them

$$\alpha \mid x \sim N(\hat{\alpha}, w_{11^*})$$

$$\theta \mid x \sim N(\hat{\theta}, w_{22^*})$$

Where w_{11^*} represents the element in position (1,1) of the matrix w^{-1} and w_{22^*} represent the element in position (2,2) of the matrix w^{-1}

Under the linex loss function, the estimates for α and θ are as follows

$$\left.\begin{aligned} \hat{\alpha}lin &= \hat{\alpha}_{sq} - \frac{bw_{11^*}}{2} \\ \hat{\theta}lin &= \hat{\theta}_{sq} - \frac{bw_{22^*}}{2} \end{aligned}\right\} \tag{21}$$

We find an important problem in the Bayesian estimation using the linex loss function, which is to choose a value for the parameter b, where the negative value of b provides a greater weight under the estimate, and when a large positive quantity, this will provide a greater weight for the above estimate. In order to make a balance in the weights between above and below estimation, Nassar,M.et al.(2022) suggested that the shape parameter should be a random variable, and in this case the balance between above and below estimation will be achieved at all possible values of the random variable They suggested three distributions, namely the generalized logistic distribution of type I, the normal and the standard Gumbel. In this paper, we choose a probability distribution for the shape parameter b described by $b \sim N(\mu, 1)$ and the probability density function b is

$$f(b) = \frac{1}{\sqrt{2\pi}}e^{\frac{-(b-\mu)^{2}}{2}} \tag{22}$$

Using the above distribution, the Elinex estimate is above

$$\hat{\alpha}_{Elin} = \int E\;\hat{\alpha}lin\;\;f(b)db \tag{23}$$

$$\hat{\theta}_{Elin} = \int E\;\hat{\theta}lin\;\;f(b)db \tag{24}$$

Using the distribution defined in eq.(no.22) in eq. (no.23,24) we get

$$\alpha_{E\,lin} = \hat{\alpha}_{sq} - \frac{\mu\,w11^{*}}{2} \tag{25}$$

$$\theta_{E\,lin} = \hat{\theta}_{sq} - \frac{\mu\,w22^{*}}{2} \tag{26}$$

We also need the expected value of the linear exponential loss function α_{Elinex} and it is found as follows

$$R_{Elin} = \iint E_\alpha \; E_b \; \left[e^{b(\hat{\alpha}_{E\,lin}-\alpha)} - b(\hat{\alpha}_{E\,lin}-\alpha) - 1\right] p(\alpha \,/\, data) \,] f(b) d\alpha \, db \qquad (27)$$

$$R_{Elin} = \iint_{-\infty}^{\infty} \left[(e^{b(\hat{\alpha}_{E\,lin}-\alpha)} - b(\hat{\alpha}_{E\,lin}-\alpha) - 1) \frac{e^{\frac{(\alpha-\hat{\alpha})^2}{2w11^*}}}{\sqrt{2\pi w11^*}} \quad d\alpha \right] \quad \frac{1}{\sqrt{2\pi}} e^{\frac{-(b-\mu)^2}{2}} db \qquad (28)$$

$$= -\mu(\hat{\alpha}_{Elin} - \hat{\alpha}) - 1 + \int_{-\infty}^{\infty} [e^{b(\hat{\alpha}_{Elin}-\hat{\alpha})} \quad e^{\frac{b^2 w_{11}^*}{2}} \; \frac{1}{\sqrt{2\pi}} e^{\frac{-(b-\mu)^2}{2}} db \,] \qquad (29)$$

Using the Maclaurian series on $e^{\frac{b^2 w_{11}^*}{2}}$ up to the first order and making simplifications and making integrations, we get

$$R_{Elin=} -\mu(\hat{\alpha}_{Elin} - \hat{\alpha}) - 1 + e^{-(\hat{\alpha}_{Elin}-\hat{\alpha})\mu + \frac{(\hat{\alpha}_{Elin}-\hat{\alpha})^2}{2}} + \frac{w_{11}^*}{2}(1 + (\mu + \hat{\alpha}_{Elin} - \hat{\alpha})^2) e^{-\frac{(\mu^2-(\mu+\hat{\alpha}_{Elin}-\hat{\alpha})^2}{2}} \qquad (30)$$

In the same way, the risk function for θ is

$$R_{Elin} = -\mu(\hat{\theta}_{Elin} - \hat{\theta}) - 1 + e^{-(\hat{\theta}_{Elin}-\hat{\theta})\mu + \frac{(\hat{\theta}_{Elin}-\hat{\theta})^2}{2}} + \frac{w_{11}^*}{2}(1 + (\mu + \hat{\theta}_{Elin} - \hat{\theta})^2) e^{-\frac{(\mu^2-(\mu+\hat{\theta}_{Elin}-\hat{\theta})^2}{2}} \qquad (31)$$

IV. SIMULATION STUDY

In this section in order to investigate and compare the performance of different Bayesian estimators using the E linex and Sq loss functions.

The data generated from TLR with sample sizes (n=25,50,100) and (α=1.5) (θ=0.5,2.5) The values of hyperparameters for the priors of α,θ are ($a_0 = 0.5, b_0 = 0.8, c_0 = 0.9, d_0 = 0.7$). Also we consider the values of shape parameter of linex loss function are (μ=-2, -1, 0.5) Bayesian estimators under E linex and Sq loss function for parameter of TLR and MSE of estimators has been evaluated and put the results in Tables (1-3) below.

TABLE 1. Bayesian estimators for α and θ under sq and Elinex loss functions at α=1.5, μ= -2 and MSE

nn	**θ**	**Quadratic**		**SE Quadrati**	**Linex**		**MSE Linex**
		$\hat{\alpha}$	$\hat{\theta}$		$\hat{\alpha}$	$\hat{\theta}$	
25	0.5	1.4713	0.4819	0.1951	1.2718	0.4677	0.1356
	2.5	1.3742	2.2314	0.473	1.2016	1.9097	0.6459
50	0.5	1.4932	0.4937	0.1019	1.3969	0.4866	0.0833
	2.5	1.4448	2.3815	0.241	1.355	2.2116	0.2732
100	0.5	1.4837	0.4946	0.0456	1.4383	0.4911	0.043
	2.5	1.4583	2.4207	0.1339	1.4144	2.3355	0.145

TABLE 2. Bayesian estimators for α and θ under sq and Elinex loss functions at α=1.5, μ= --1 and MSE

n	**θ**	**Quadratic**		**MSE Quadrati**	**Linex**		**MSE linex**
		$\hat{\alpha}$	$\hat{\theta}$		$\hat{\alpha}$	$\hat{\theta}$	
25	0.5	1.4723	0.4887	0.1707	1.3737	0.4814	0.1362
	2.5	1.3479	2.216	0.4589	1.2664	2.0559	0.5195
50	0.5	1.4678	0.4909	0.0901	1.422	0.4873	0.0825
	2.5	1.4275	2.3505	0.2685	1.3837	2.2671	0.2786
100	0.5	1.5021	0.5006	0.0527	1.4787	0.4988	0.0493
	2.5	1.4624	2.4233	0.1322	1.4403	2.3806	0.1349

It is seen from tables (1,2) above The E linex estimators is better than Sq estimators except for the case of θ=2.5 and all sample sizes

TABLE 3. Bayesian estimators for α and θ under sq and Elinex loss functions at α=1.5, μ= 0.5 and MSE

n	θ	Quadratic		MSEQuad	Linex		MSE Linex
		$\hat{\alpha}$	$\hat{\theta}$		$\hat{\alpha}$	$\hat{\theta}$	
25	0.5	1.4928	0.4924	0.1935	1.5443	0.496	0.2308
	2.5	1.3574	2.2168	0.4303	1.3985	2.2962	0.4271
50	0.5	1.4949	0.4959	0.098	1.519	0.4977	0.1062
	2.5	1.4434	2.3693	0.2484	1.4659	2.4114	0.2529
100	0.5	1.4976	0.496	0.0471	1.5092	0.4968	0.0489
	2.5	1.4726	2.4352	0.129	1.4838	2.4567	0.1301

It is seen from tables (3) above that Bayesian estimators under Sq is better E linex Except for the case of θ=2.5, n=25

V. REAL DATA ANALYSIS

In this section the data set represent the time of infection of kidney dialysis patients in month taken from Klein and Moesch erger (2006)The goodness of fit kolmogrov smirnov (K.S.)and Chi-square goodness of fits statistic is used to fit real data set we put the hypothesis H0:Data follow TLR r.s H1: :Data follow Doesnt TLR
The results of test statistics in table (4) below

TABLE 4. K.S. and Chi-square tests of fitting data by TLR distribution

Test statistic	Calculated values		Tabulated value
K.S.	0.143	<	0.263
Chi-square	0.863	<	3.8

From table (4), it is concluded the null hypothesis is accepted Bayesian estimators for parameters of TLR under Sq and the E linex which defined in eq. (no.30) three different values for the mean of shape parameter(b) of linex loss function are (μ=-2, -1,0.5) The results are in table (5) below

TABLE 5. Bayesian estimators under linex and Sq loss function and Bayesian risks for time of infection data in months

Methods		$\hat{\alpha}$	$\hat{\theta}$	RQ
Quadratic		0.7082	0.00213	0.02228
RElin	$\mu = -2$	0.68593	0.00213	0.02073
	$\mu = -1$	0.69706	0.00213	0.0215
	$\mu = 0.5$	0.71377	0.00213	0.02267

From table above the best estimators for parameters of TLR are
$\hat{\alpha}_{Elinex=0.68593}$,$\hat{\theta}_{Elinex=0.00213}$ with $R_{Elinex=0.02073}$

VI. CONCLUSIONS

Rayleigh distribution generalized by TL family This new distribution named as TLR It is a positive skew distribution. Some statistical properties of the distribution has been found such as r-th moment around zero, mode, quantile and moment generating functions. The parameters of distribution estimated by Bayesian technique under Sq and E linex loss functions The results of estimation applied in simulated and real data sets, The important conclusion in simulated data set that the Elinex are best estimators when the mean of shape parameter of E linex function less than zero and scale parameter of TLR less than 2.5. Also the estimators under Sq is the best estimator when μ>0 except the case of scale parameter of TLR equal to 2.5 In real data set it is concluded that the Elinex estimator when the location parameter μ equal -2 is the best.

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التقدير البيزي لمعلمات توزيع توب ليون رايلي تحت دالتي الخسارة التربيعية والاسية الخطية مع التطبيق

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الخلاصة: في هذه الورقة درست الخصائص الإحصائية لتوزيع توب ليون رايلي وتشمل العزم حول الصفر والمنوال والوسيط ودالة الربيعات والتفلطح والالتواء والدالة المولدة للعزم، الطريقة البيزية باستخدام التوزيع الأولي لكاما وتقدير معلمات التوزيع تحت دالتي الخسارة التربيعية والأسية الخطية. معلمة الموقع (b) لدالة الخسارة الأسية الخطية تعامل كمتغير عشوائي واستخدم $b\sim N\,(M,1)$. طبقت النتائج في مجموعة البيانات الحقيقية والمحاكاة. في بيانات المحاكاة ولجميع أحجام العينات المولدة من توزيع (TLR) ولقيم مختلفة لمعلمات التوزيع وقيم مختلفة، لـ μ ، التقدير تحت دالة الخسارة الأسية الخطية أفضل عندما معلمة الموقع (μ <0) . ما عدا في حالة معلمة القياس θ =2.5 . والتقدير تحت دالة الخسارة التربيعية أفضل عندما (μ > 0) وحجم العينة n = 25 . في مجموعة البيانات الحقيقية التقدير تحت دالة الخسارة الأسية الخطية أفضل عندما (μ= −2) النتائج للبيانات الحقيقة مساندة لنتائج بيانات المحاكاة.

الكلمات المفتاحية: عائلة توب ليون رايلي، التوزيع الاولي، توزيع رايلي، دالة الخسارة التربيعية، دالة الخسارة الاسية الخطية، دالة الخسارة الاسية الخطية المتوقعة.