

Iraqi Journal of Statistical Sciences



http://stats.uomosul.edu.iq

CUSUM Control Chart for Symlets Wavelet to Monitor Production Process Quality

Duaa Faiz Abdullah¹

Jwana Rostom Qadir²

Diyar Lazgeen Ramadhan³

and Taha Hussein Ali⁴

- ^{1,4} Department of Statistics and Informatics, College of Administration and Economics, University of Salahaddin, Erbil, Kurdistan Region, Iraq
- ² Raparin University- Ranya, Kurdistan Region, Iraq, College of Science, Department of Medical Laboratory.
- ³ Duhok University, College of Health Sciences, Department of Medical Laboratory Sciences, Duhok, Iraq

Article information

Article history:

Received April 17, 2024 Revised: September 20,2024 Accepted: September 26, 2024 Available online December 1, 2024

Keywords:

CUSUM Control Chart, Tabular method, De-noise data, Symlets Wavelet, Wavelet Shrinkage.

Correspondence: Taha Hussein Ali taha.ali@su.edu.krd

Abstract

In this paper, it was proposed to create a new chart that represents the Symlets wavelet chart with orders of (1, 2, and 3) to obtain discrete wavelet transformation maximum overall coefficients, through which the threshold parameter is estimated using the universal method and then applied hard threshold rule to obtain de-noise data which will be relied upon in constructing the Cumulative Sum Chart using the Tabular method. The efficiency of the proposed chart was measured and compared to the classical chart by simulating several cases and real data and calculating the difference between the control limits (Difference), the standard deviation and the number of points outside the control limits to determine the sensitivity of the chart to minor changes that may occur in the production process, using an algorithm in the MATLAB program it was designed for this purpose. The research results revealed that the proposed charts are more efficient and sensitive to minor changes (that may event) in the production process than the classical charts.

DOI: 10.33899/iqjoss.2024.185240, @Authors, 2024, College of Computer Science and Mathematics, University of Mosul. This is an open access article under the CC BY 4.0 license (http://creativecommons.org/licenses/by/4.0/).

1. Introduction

In today's competitive manufacturing landscape, ensuring consistent product quality is essential for maintaining competitiveness and meeting customer expectations. Traditional quality control methods, including the Cumulative Sum (CUSUM) chart, are valuable tools for identifying process deviations. However, in phase I of increasingly complex and variable production settings, these techniques may struggle to detect subtle aberrations or noise in the data [1]. CUSUM charts are particularly effective for detecting small shifts in process means that may not be easily detectable using traditional control charts like the Shewhart control chart [2]. There are different variations of CUSUM charts, including one-sided CUSUM and two-sided CUSUM, depending on whether you are interested in detecting shifts in one direction or both directions. Additionally, parameters such as the reference value, decision intervals, and smoothing constants need to be determined when constructing a CUSUM chart [3]. By using the tabular method, you can track the cumulative sums and make decisions based on the values directly from the table, rather than relying on graphical interpretation from a chart. This can be particularly useful in situations where a visual chart is not practical or when you prefer a more straightforward method of analysis [4]. De-noising data involves removing or reducing unwanted noise or disturbances from a dataset while retaining meaningful information [5]. The choice of de-noising

method depends on the characteristics of the data and the nature of the noise present. It's often beneficial to experiment with different methods and parameters to find the most suitable approach for a particular dataset [6]. A possible approach to overcoming these obstacles is wavelet analysis. Wavelet analysis is a powerful technique that breaks down signals into their constituent frequency components to extract meaningful information while removing noise [7]. Symlets are unique among wavelet families in that they can balance frequency localization and vanishing moments, which makes them perfect for applications involving feature extraction and signal denoising [8]. The CUSUM control chart and Symlets wavelet analysis are used in this article to provide a novel way to improve the monitoring of production process quality [9]. We want to get around the drawbacks of traditional quality control procedures by utilizing state-of-the-art signal processing techniques. We employ Symlets wavelets, specifically Symlets 1, 2, and 3, to effectively reduce noise in production data and retrieve valuable information while maintaining essential attributes necessary for assessing quality. The suggested method integrates Symlets wavelets (orders 1, 2, and 3) with wavelet shrinkage to obtain de-noised data. Noise is efficiently eliminated using a hard threshold rule in conjunction with discrete wavelet treatment [10]. The Tabular technique is then used to create a CUSUM chart utilizing the de-noised data that have been produced. We anticipate substantial improvements in monitoring production process quality efficiency through this integrated approach. By promptly identifying deviations and implementing proactive quality control measures, we aim to enhance product quality and improve the costeffectiveness and efficiency of manufacturing processes.

2. Quality Control Charts

Quality control charts, vital in manufacturing and process industries, monitor process variability over time. They distinguish between common cause and special cause variations, aiding timely intervention [11]. Simple and intuitive, they plot sample statistics against control limits. Charts like X-bar, R-chart, and p-chart are commonly used. They aid process improvement by revealing trends and patterns. However, they may struggle with subtle deviations and rely on assumptions like normality. Enhancements include advanced statistical techniques and computational tools. In subsequent sections, we explore the Cumulative Sum (CUSUM) control chart and its integration with wavelet analysis for improved process monitoring and quality assurance [12].

2.1. CUSUM Control Chart

CUSUM Control Charts are a powerful tool for detecting subtle shifts in process performance, offering greater sensitivity compared to traditional control charts. Introduced by E.S. Page in 1954, CUSUM charts track the cumulative sum of deviations from a target value over time. They are more effective at identifying changes in process parameters quickly and can detect minor deviations that may be missed by other methods [13]. The flexibility of CUSUM charts extends to various process types and data distributions, making them resilient and customizable. Different versions, such as one-sided, two-sided, and exponentially weighted moving average (EWMA) CUSUM charts, cater to different needs. Advanced versions like multivariate and adaptive CUSUM charts have also been developed for more complex scenarios [14]. Overall, CUSUM control charts enhance proactive process monitoring and control, ultimately leading to improved product quality and operational efficiency. While their visual approach, known as the V-Mask method, aids in understanding process behaviour, the Tabular method provides a simpler alternative [15].

2.2. The Tabular method

Instead of presenting the cumulative sums of deviations from a target value on a graph, the Tabular approach for CUSUM control charts computes and records them in a table format. For individuals who prefer a more direct approach to data interpretation, this method offers a more basic method for numerically analysing process data. Here are the steps to make a CUSUM chart [16]. Estimate the standard deviation of the data from the moving range control chart $\sigma = \overline{R}/d_2$. For univariate (r=2), $d_2=1.128$

- Calculate the reference value or allowable slack since the CUSUM chart monitors the small shifts. Generally, 0.5 to 1 sigma will be considered. $k = 0.5 \sigma$.
- Compute decision interval H, generally $\pm 4 \sigma$ will be considered (some place $\pm 5 \sigma$ also be used).
- Calculate the upper and lower CUSUM values for each individual i value.
- Upper CUSUM (UC_i) = Max $[0, UC_{i-1}+x_i Target value-k)$.

- Lower CUSUM (LC_i) = Min [0, LC_{i-1}+x_i Target value+k).
- Draw all UC_i & LC_i values in the graph and also draw decision intervals (UCL and LCL).
- Check if any of the UCi values are above the UCL and any of the LCi values are below the LCL.
- Finally, take necessary action to eliminate the special causes if any of the points are out of control limits.

To find the plotting position for the new point in a cumulative sum chart, you can use the following formula.

$$S_n = k + \frac{1}{\sigma_x} \sum_{i=1}^n (x_i - T) \tag{1}$$

 S_n is the plotting position, k is the reference value or the center line of the cumulative sum chart (if provided), x_i represents the new point you want to plot, T is the Target value and σ_x is the Standard Deviation.

3. Wavelet Shrinkage

Wavelet shrinkage is a technique used for signal denoising based on wavelet transforms. It involves shrinking or thresholding the wavelet coefficients to remove noise while preserving the essential features of the signal [17]. However, wavelet shrinkage also has limitations, such as the need to select appropriate thresholding parameters and the potential for introducing artefacts if the thresholding is not performed carefully. Additionally, wavelet shrinkage may not perform well for signals with complex noise structures or non-Gaussian noise [18].

3.1 Symlets Wavelet

Symlets (short for Symmetric Least Asymmetric) wavelet is a family of wavelets used in signal processing and data compression. They are similar to Daubechies wavelets but have more symmetry and fewer vanishing moments [19]. The Symlets (Sym) are variations on the (db) family of wavelets developed by Daubechies that are approximately symmetrical, orthogonal, and biorthogonal. The two wavelet families' characteristics are the same. Sym N has N as its order [20].

3.2 Universal Thresholding

The universal threshold approach provided by [21] is shown by the following formula.

$$\delta^{U} = \hat{\sigma}_{MAD} \sqrt{2 \log \log (n)} \tag{2}$$

Where $\hat{\sigma}_{MAD}$ is the standard deviation estimator of details coefficients, and is equal to $\frac{MAD}{0.6745}$. The wavelet coefficients' median absolute deviation at the finest scale is known as MAD [22].

3.3 Hard Threshold Rule

Hard thresholding, a straightforward approach for thresholding and evaluating the proposition of (keep or kill), was proposed by Donoho and Johnstone. Wavelet de-noising was implemented by a simple approach called Hard Thresholding [23]. The wavelet coefficient is set to the vector $Wn^{(H)}$ with element.

$$Wn^{(H)} = \begin{bmatrix} 0 & if & |Wn| \le \delta \\ Wn & if & |Wn| > \delta \end{bmatrix}$$
(3)

Exceeding δ are left untouched, while smaller than or equal to δ are eliminated or set to 0. Thus, the operation of hard thresholding is not continuous mapping.

4. Proposed Chart

To address data noise and configure the proposed chart, the following steps can be performed:

- Transforming data using Symlets wavelets: Symlets wavelets with different orders (1, 2, and 3) transform the original data to discrete wavelet transformation coefficients.

- Estimating the threshold level using the Universal method:
 The Universal method is used to determine the threshold level to be used in the rasterization process [24].
- Application of hard rule:
 A hard threshold rule is applied to the maximum overlap discrete wavelet transformation coefficients to refine denoise data.
- De-Noise data setting:

 After cleaning the data using a hard threshold rule, de-noise data can be used in creating the proposed chart.
- Create a CUSUM chart using the Tabular method: The proposed chart employs wavelet shrinkage in data processing to obtain de-noise data through the use of Symlets wavelet with orders of (1, 2, and 3) to obtain maximum overlap discrete wavelet transformation coefficients, through which the threshold parameter is estimated using the universal method and then applied hard threshold rule to obtain de-noise data which will be relied upon in constructing the CUSUM chart using the Tabular method.

5. Application aspect

To compare the classical and the proposed charts in terms of efficiency and accuracy of the estimated parameters (the average moving range (AMR) and σ) and the difference between upper and lower control limits, in addition to calculating the number of points outside the limits of control to determine the sensitivity of the proposed charts to minor changes that may occur in the production process, the simulation study was done by simulating the CUSUM quality control chart, then the application for the real data. By designing a program in MATLAB (version 2022a) dedicated to this purpose (Appendix).

5.1. Simulation study

To generate data for the CUSUM chart, the standard uniform distribution function was used plus linearly equally spaced points between (0 and 0.5) and (0 and 1) for different samples (m=20 and 25) observations. The data generation was also repeated (10,000) times, through which the classical CUSUM charts were formed using the Tabular method (r=2, $d_2=1.128$, k=0.5) and then the proposed charts were formed based on Symlets wavelets of orders (1, 2 and 3) with Universal de-noising method and hard threshold rule. The average of the quality control limits (lower and upper limits), the average moving range (AMR), the difference between the control limits (Difference), the standard deviation (Sigma) and the number of points outside the control limits are summarised in Tables (1-4).

Table 1. Average simulation results at m = 20 and interval (0, 0.5)

Chart	LCL	UCL	AMR	Difference	Sigma	No. of Point
Classical	-1.1843	1.1843	0.3340	2.3687	0.2961	0.4843
Sym1	-0.4394	0.4394	0.1239	0.8788	0.1098	5.2791
Sym2	-0.5666	0.5666	0.1598	1.1332	0.1417	3.6841
Sym3	-0.5938	0.5938	0.1674	1.1875	0.1484	3.4662

Table 2. Average simulation results at m = 25 and interval (0, 0.5)

g							
	Chart	LCL	UCL	AMR	Difference	Sigma	No. of Point
	Classical	-1.1853	1.1853	0.3343	2.3706	0.2963	0.7716
	Sym1	-0.4848	0.4848	0.1367	0.9696	0.1212	6.5752
	Sym2	-0.5615	0.5615	0.1583	1.1229	0.1404	5.2811
	Sym3	-0.5490	0.5490	0.1548	1.0980	0.1372	5.5167

Table 3. Average simulation results at m = 20 and interval (0, 1)

Chart	LCL	UCL	AMR	Difference	Sigma	No. of Point
Classical	-1.1915	1.1915	0.3360	2.3831	0.2979	2.0812
Sym1	-0.4566	0.4566	0.1288	0.9132	0.1141	6.4098
Sym2	-0.5814	0.5814	0.1639	1.1627	0.1453	5.3738
Sym3	-0.6078	0.6078	0.1714	1.2157	0.1520	5.2191

Table 4. Average simulation results at m = 25 and interval (0, 1)

Chart	LCL	UCL	AMR	Difference	Sigma	No. of Point
Classical	-1.1898	1.1898	0.3355	2.3797	0.2975	3.2487
Sym1	-0.4952	0.4952	0.1396	0.9904	0.1238	8.1262
Sym2	-0.5707	0.5707	0.1609	1.1414	0.1427	7.3191
Sym3	-0.5585	0.5585	0.1575	1.1170	0.1396	7.4617

Table .1-4 shows that all the proposed charts were better than the classical chart, depending on the difference, Sigma, which has lower values than its counterpart in the classical method, in addition, the average of points outside the control limits was greater than its counterparts, which indicates the sensitivity of the proposed chart in detecting minor changes that may occur in the production process. The first proposed chart (Sym1) was the best compared to the rest of the proposed charts for all simulation cases. Increasing the number of observations (from 20 to 25) and the interval ((0, 0.5) to (0, 1)) leads to an increase in the difference and Sigma as well as an increase in the number of points falling outside the control limits.

5.2. Real data

CUSUM chart will be used in the control phase of DMAIC (Define, Measure, Analyze, Design, and Verify). DMAIC is a data-driven quality strategy used to improve processes. It is an integral part of a Six Sigma initiative, in general, it can be implemented as a standalone quality improvement procedure or as part of other process improvement initiatives. In a drug manufacturing unit, potassium content is one of the important parameters. The target potassium content is 0.21% in weight (wt) [25]. Therefore, the team has collected 15 batches from the production in a time interval to monitor the process mean shift, are shown in Table 5.

Table 5. Real data for Potassium 0.21% in wt

Sample	1	2	3	4	5
Potassium 0.21% in wt	0.235	0.222	0.198	0.227	0.199
Sample	6	7	8	9	10
Potassium 0.21% in wt	0.242	0.206	0.196	0.221	0.212
Sample	11	12	13	14	15
Potassium 0.21% in wt	0.231	0.246	0.249	0.245	0.255

The original data for potassium 0.21% in wt and then the de-noise data using Symlets wavelets of orders (1, 2 and 3) with Universal de-noising method and hard threshold rule were plotted in Figure 1.

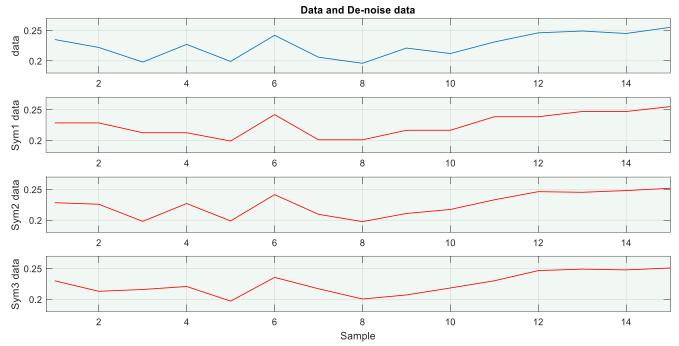


Figure 1. Real data and de-noise data

The quality control limits (lower and upper limits), the average moving range (AMR), the difference between the control limits (Difference), the standard deviation (Sigma) and the number of points outside the control limits for the classical and proposed method are summarised in Table 6.

Table 6. Results of real data

Chart	LCL	UCL	AMR	Difference	Sigma	No. of Point
Classical	-0.0679	0.0679	0.0191	0.1358	0.0170	3
Sym1	-0.0424	0.0424	0.0120	0.0849	0.0106	4
Sym2	-0.0584	0.0584	0.0165	0.1168	0.0146	3
Sym3	-0.0444	0.0444	0.0125	0.0889	0.0111	4

Table 6 shows that all the proposed charts were better than the classical chart, depending on the difference, Sigma, which has lower values than its counterpart in the classical method, in addition, the average of points outside the control limits was greater than its counterparts, which indicates the sensitivity of the proposed chart in detecting minor changes that may occur in the production process. The first proposed chart (Sym1) was the best compared to the rest of the proposed charts for real data. The points and quality control limits on the classical and proposed (Sym1) charts are plotted in Figures 2 and 3.

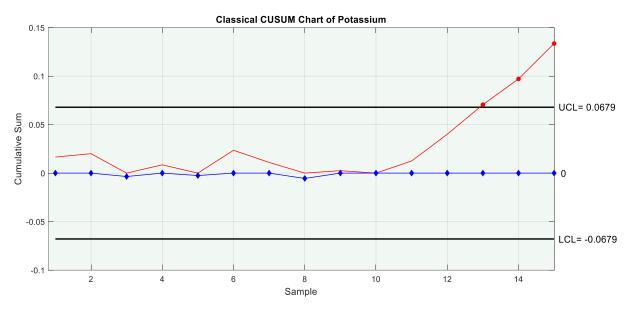


Figure 2. Classical CUSUM Chart for Real Data

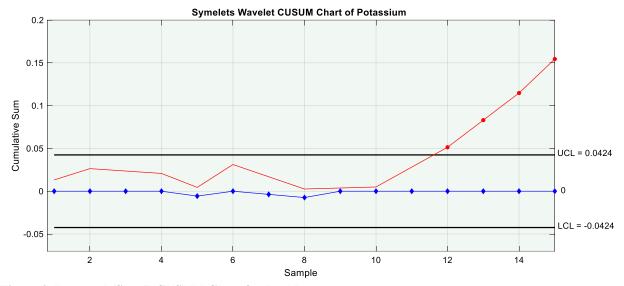


Figure 3. Proposed (Sym1) CUSUM Chart for Real Data

It is evident from Figure 2 (classical chart) that from sample 13 onwards the process is going out of control (3 points) while from Figure 3 (proposed (Sym1) chart) from sample 12 onwards the process is going out of control (4 points). Hence, the proposed (Sym1) chart has more sensitivity than the classical chart in detecting minor changes that may occur in the production process.

6. Conclusion & Recommendations

Through the study of simulation and real data, the following main conclusions and recommendations were summarized:

6.1. Conclusions

- 1. All the proposed (Sym1, 2, and 3) charts were better than the classical chart for the CUSUM chart for all simulation cases and real data.
- 2. The proposed charts have more sensitivity than the classical CUSUM chart in detecting minor changes that may occur in the production process.

3. The first proposed chart (Sym1) was the best compared to the rest of the proposed charts for all simulation cases and real data.

6.2. Recommendations

- 1. Using the proposed chart for Quality Control of the CUSUM.
- 2. Conducting a prospective study on the use of the Symlets Wavelet with a multivariate CUSUM chart.
- 3. Conducting a prospective study on the use of the other wavelets with a CUSUM chart.

Conflict of interest

The author has no conflict of interest.

References

- 1. Koshti, V. V. (2011). Cumulative Sum Control Chart, International Journal of Physics and Mathematical 32.
- 2. Ali, Taha Hussein, Rahim, Alan Ghafur, and Saleh Dlshad Mahmood, "Construction of Bivariate F-Control Chart with Application" Eurasian Journal of Science & Engineering, 4.2 (2018): 116-133.
- 3. C. P. Quesenberry, "The multivariate short-run snapshot Q chart," Quality Engineering, vol. 13, no. 4, pp. 679–683, 2001.
- 4. Montgomery, D. C. (2012). Introduction to statistical quality control. John Wiley & Sons. http://eu.wiley.com/WileyCDA/WileyTitle/productCd-EHEP002023.html.
- Ali, Taha Hussein, Nasradeen Haj Salih Albarwari, and Diyar Lazgeen Ramadhan. "Using the hybrid proposed method for Quantile Regression and Multivariate Wavelet in estimating the linear model parameters." Iraqi Journal of Statistical Sciences 20.1 (2023): 9-24.
- 6. Omer, A. W., Sedeeq, B. S., & Ali, T. H. (2024). A proposed hybrid method for Multivariate Linear Regression Model and Multivariate Wavelets (Simulation study). Polytechnic Journal of Humanities and Social Sciences, 5(1), 112-124.
- 7. Ali, Taha Hussein & Mardin Samir Ali. "Analysis of Some Linear Dynamic Systems with Bivariate Wavelets" Iraqi Journal of Statistical Sciences 16.3 (2019): 85-109.
- 8. Cabana, E. and Lillo, R. E. (2021). Robust multivariate control chart based on shrinkage
- 9. Ali, Taha Hussein, and Saleh, Dlshad Mahmood, "Proposed Hybrid Method for Wavelet Shrinkage with Robust Multiple Linear Regression Model: With Simulation Study" QALAAI ZANIST JOURNAL 7.1 (2022): 920-937.
- 10. Samad Sedeeq, B., Muhammad, Z. A., Ali, I. M., & Ali, T. H. (2024). Construction Robust -Chart and Compare it with Hotelling's T2-Chart. Zanco Journal of Human Sciences, 28(1), 140–157. https://doi.org/10.21271/zjhs.28.1.11.
- 11. Grasso, M., Menafoglio, A., Colosimo, B. M., and Secchi, P. (2016). Using curve-registration information for profile monitoring. Journal of Quality Technology, 48(2):99–127.
- 12. Centofanti, F., Colosimo, B. M., Grasso, M. L., Menafoglio, A., Palumbo, B., and Vantini, S. (2021a). Robust functional ANOVA with application to additive manufacturing.
- 13. Chenouri, S., Steiner, S. H., and Variyath, A. M. (2009). A multivariate robust control chart for individual observations. Journal of Quality Technology, 41(3):259–271.
- H. R. Moheghi, R. Noorossana & O. Ahmadi (2022) Phase I and phase II analysis of linear profile monitoring using robust estimators, Communications in Statistics - Theory and Methods, 51:5, 1252-1269, DOI: 10.1080/03610926.2020.1758724.
- 15. Kareem, Nazeera Sedeek and Mohammad, Awaz Shahab, and Ali, Taha Hussein, "Construction robust simple linear regression profile Monitoring" Journal of Kirkuk University for Administrative and Economic Sciences, 9.1. (2019): 242-257.
- 16. Johnson, R. A., Wichern, D. W., et al. (2014). Applied multivariate statistical analysis, volume 6. Pearson London, UK.

- 17. Ali, Taha Hussein and Jwana Rostam Qadir. "Using Wavelet Shrinkage in the Cox Proportional Hazards Regression model (simulation study)", Iraqi Journal of Statistical Sciences, 19, 1, 2022, 17-29.
- 18. Mustafa, Qais, and Ali, Taha Hussein. "Comparing the Box Jenkins models before and after the wavelet filtering in terms of reducing the orders with application." Journal of Concrete and Applicable Mathematics 11 (2013): 190-198.
- 19. Ali, Taha Hussein, and Saleh, Dlshad Mahmood, "Proposed Hybrid Method for Wavelet Shrinkage with Robust Multiple Linear Regression Model: With Simulation Study" QALAAI ZANIST JOURNAL 7.1 (2022): 920-937
- 20. Ali, Taha Hussien, and Nazeera Sedeek Kareem. "Awaz shahab mohammad, (2021), Data de-noise for Discriminant Analysis by using Multivariate Wavelets (Simulation with practical application)." Journal of Arab Statisticians Union (JASU) 5: 78-87.
- 21. Donoho, David L., and Iain M. Johnstone. "Ideal Spatial Adaptation by Wavelet Shrinkage." Biometrika, vol. 81, no. 3, 1994, pp. 425–55. JSTOR, https://doi.org/10.2307/2337118. Accessed 18 Mar. 2024.
- 22. Ali, Taha Hussein, and Saleh, Dlshad Mahmood. "Comparison Between Wavelet Bayesian and Bayesian Estimators to Remedy Contamination in Linear Regression Model" PalArch's Journal of Archaeology of Egypt/Egyptology 18.10 (2021): 3388-3409.
- 23. Hagiwara, Kenta, Satoshi Horikoshi, and Nick Serpone. "Photoluminescent carbon quantum dots: synthetic approaches and photophysical properties." Chemistry—A European Journal 27.37 (2021): 9466-9481.
- 24. Sakar Ali Jalal; Dlshad Mahmood Saleh; Bekhal Samad Sedeeq; Taha Hussein Ali. "Construction of the Daubechies Wavelet Chart for Quality Control of the Single Value". IRAQI JOURNAL OF STATISTICAL SCIENCES, 21, 1, 2024, 160-169. doi: 10.33899/iqjoss.2024.183257
- 25. Umble, Elisabeth J. "Cumulative Sum Charts and Charting for Quality Improvement." Technometrics 43 (2001): 107 107.

Appendix

لوحة سيطرة المجموع المتراكم للموبجة سيملت لمراقبة جودة عملية الإنتاج

دعاء فائز عبد الله، جوانه روستم قادر، ديار لازكين رمضان، طه حسين علي

^{1 ، 4} قسم الإحصاء والمعلوماتية ، كلية الإدارة والاقتصاد ، جامعة صلاح الدين ، أربيل ، إقليم كردستان ، العراق ، أحمعة رابارين—رانيا ، إقليم كردستان ، العراق ، كلية العلوم ، قسم المختبرات الطبية ، دهوك ، العراق . العراق . العراق . كلية العلوم الصحية ، قسم علوم المختبرات الطبية ، دهوك ، العراق . المخلاصة: في هذا البحث تم اقتراح إنشاء لوحة جديدة تمثل لوحة المويجات سيملت بالرتب (1 ، 2 ، 3) للحصول على معاملات التحويل المتقطع للمويجات ذات الفائض العال ، والتي من خلالها يتم تقدير معلمة قطع العتبة باستخدام الطريقة الشاملة ومن ثم طبقت قاعدة قطع العتبة الصلبة للحصول على بيانات ذات ضوضائية أقل والتي سيتم الاعتماد عليها في إنشاء لوحة المجموع التراكمي باستخدام الطريقة الجدولية. تم قياس ومقارنة كفاءة اللوحة المقترحة مع اللوحة التقليدية من خلال محاكاة عدة حالات وبيانات حقيقية وحساب الغرق بين حدود السيطرة والانحراف المعياري وعدد النقاط الواقعة خارج حدود السيطرة لتحديد حساسية اللوحة للتغييرات الطفيفة التي قد تحدث في عملية الإنتاج، وذلك باستخدام خوارزمية مصممة لهذا الغرض في برنامج ماتلاب.

الكلمات المفتاحية: لوحة سيطرة المجموع المتراكم، الطريقة الجدولية، تقليص ضوضائية البيانات، المويجة سيملت، التقليص المويجي.

أظهرت نتائج البحث أن اللوحات المقترحة أكثر كفاءة وحساسية للتغيرات الطفيفة (التي قد تحدث) في عملية الإنتاج من اللوحات التقليدية.