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Eigen value for image segmentation

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ABSRACT

In this paper eigenvalue of a gray image is used for image segmentation by calculating the eigenvalue for each segment and to be compared with a threshold value [which will be calculated dynamically by the system] and an iteration procedure will continue running until saturation arrived.

By using the quadtree technique the gray image is divide into small segments depending on the value of the threshold and then the segmented image will pass into low pass filter in order to produce more enhancement in the image .

The algorithm was applied on different types of images [natural images , text images] and both of them give high quality of segmentation .

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تقطيع الصور الرقمية باستخدام القيم المميزة

الملخص

في هذا البحث تم اعتماد القيمة المميزة(eigen value) للصور الرمادية من اجل التقطيع وذلك من خلال حساب القيمة المميزة لكل جزء، ومن ثم مقارنته مع قيمة حد العتبة (الذي يتم حسابه ذاتيا من خلال النظام) وتستمر العملية الى حد الوصول الى حالة الاستقرار.

اعتماد التقسيم الشجري الرباعي من اجل تقسيم الصورة الرمادية الى اجزاء صغيرة اخذا بنظر الاعتبار حد العتبة ، بعدها يتم امرار الاجزاء المقسمة داخل مرشح واطئ التردد لاجل الحصول على تحسين اعلى للصور.

الخوارزمية تم تطبيقها على انواع مختلفة من الصور(صور طبيعية ،صور نصية) وكلاهما تم الحصول على تقطيع بمواصفات جيدة.

1.Introduction:

One of the most significant problems in object recognition is to find the important features of an image. Successful identification of the features can be helpful in various fields including scientific research(1) .

A segmentation could be used for an object recognition, occlusion boundary estimation within motion or stereo systems, image compression, image editing, or image database look-up(2) .

The object segmentation is a process of grouping parts of an image into units (classes, regions, subsets) that are homogeneous with respect to one or more characteristics (or features).

The most commonly used segmentation techniques can be classified into two broad categories: (a) region extraction techniques that look for maximal regions satisfying some homogeneity criterion, (b) edge extraction techniques that look for edges occurring between regions with different characteristics.

Threshold is a common region extraction method. It is based on the assumption that the image has a bimodal histogram and, therefore, contains an object or objects of interest that can be extracted from the background by a simple operation that compares image values with a threshold level.

There are several threshold methods: global methods which based on gray level histograms, global methods based on local properties, local threshold selection, and dynamic thresholding(3)

One-parameter perturbation theory of eigen values for nonsymmetric matrices and differential operators was developed in [Vishik and Lyusternik (I960)], and a constructive method for determining leading terms in eigenvalue expansions was given by [Lidskii (1965)]. These works study regular types of bifurcations, when perturbation satisfies a specific nondegeneracy condition. For the analysis of some non-regular cases see [Moro et al. (1997)]. The multi-parameter bifurcation theory for eigenvalues of nonsymmetric matrices was developed in [Seyranian (1990a), (1991a), (1993a), (1994a); Mailybaev and Seyranian (1999b); Mailybaev and Seyranian (2000a); Seyranian and Kirillov (2001)], where perturbations along different directions or curves in the parameter space were studied. Recent achievements of the theory of interaction of eigenvalues in multi-parameter problems are given in [Kirillov and Seyranian (2002a); Seyranian and Mailybaev (2003)][(4)] .

2. Aim of the paper :-

In this paper, the proposed image segmentation method is used with eigenvalue . The adopted procedure will divide the image into four segments with equal sizes then calculate the eigenvalue for each part and then will choose the minimum value to be used as threshold in quadtree segmentation and the process of image segmentation will work to locate an objects in the gray image ,the resultant image will pass via low pass filter, which makes it smooth.

3.Realated work

No searches are found in books and Internet about using the eigenvalue with segmentation for the current related work until now.

4. Image segmentation

In computer vision, **segmentation** refers to the process of partitioning a digital image into multiple segments (sets of pixels) (Also known as super pixels). The goal of segmentation is to simplify and/or change the representation of an image into something that is more meaningful and easier to analyze. Image segmentation is typically used to locate objects and boundaries (lines, curves, etc.) in images. More precisely, image segmentation is the process of assigning a label to every pixel in an image such that pixels with the same label share certain visual characteristics. The result of image segmentation is a set of segments that collectively cover the entire image, or a set of contours extracted from the image (see edge detection). Each of the pixels in a region are similar with respect to some characteristic or computed property, such as color, intensity, or texture. Adjacent regions are significantly different with respect to the same characteristic [5][6].

One of the method used in image segmentation is quadtree

5. Quadtree

A *quadtree* is a tree whose nodes are either leaves or have 4 children. The children are ordered 1, 2, 3, 4.

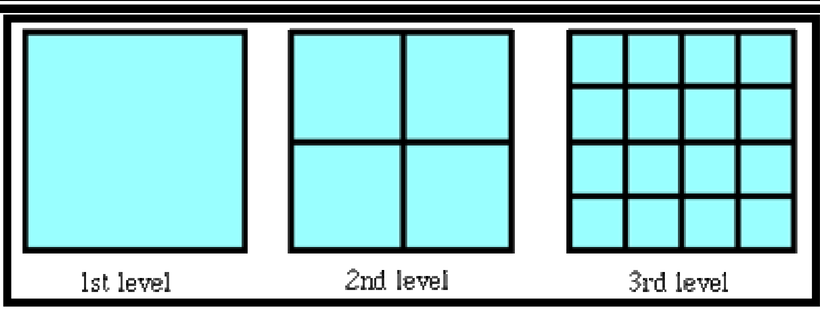
5.1. Quadtree Strategy

Here we will outline the strategy behind using quadtrees as a data structure for pictures. The key is to "Divide and Conquer".Let's say we will divide the picture area into 4 sections. Those 4 sections are then further divided into 4 subsections. This process will continue, repeatedly dividing a square region by 4. We must impose a limit to the levels of division otherwise we could go on dividing the picture forever, see that in figure(1). Generally, this limit is imposed due to the storage considerations or to limit processing time or due to the resolution of the output device. A *pixel* is the smallest subsection of the quadtree.

To summarize, a square or *quadrant* in the picture is either:

a) entire one colour.
b) composed of 4 smaller sub-squares

In terms of a quadtree, the children of a node represent the 4 quadrants. The root of the tree is the entire picture.



Figure(1) First three levels of a quadtree

To represent a picture using a quadtree, each leaf must represent a uniform area of the picture. If the picture is black and white, we only need one bit to represent the colour in each leaf; for example, 0 could mean black and 1 could mean white[(7)(8)] .

5.2. Advantages and Disadvantages of Quadtree Picture Representation

Recursive pictures can be implemented easily using quadtrees: the root of the quadtree has four children, where one of the children is the actual image and the other three point to the root.

Other advantages of quadtrees include:

- Erasing a picture takes only one step. All that is required is to set the root node to neutral.
- Zooming to a particular quadrant in the tree is a one step operation.
- To reduce the complexity of the image, it suffices to remove the final level of nodes.
- Accessing particular regions of the image is a very fast operation. This is useful for updating certain regions of an image, perhaps for an environment with multiple windows.

There are many ways of compacting quadtrees, which is important for transferring data efficiently[(8)(9)] .

In matlab the Quadtree decomposition is an analysis technique that involves subdividing an image into blocks that are more homogeneous than the image itself. This technique reveals information about the structure of the image. It is also useful as the first step in adaptive compression algorithms. You can perform quadtree decomposition using the qtdecomp function. This function works by dividing a square image into four equal-sized square blocks, and then testing each block to see if it meets some criterion of homogeneity (e.g., if all the pixels in the block are within a specific dynamic range). If a block meets the criterion, it is not divided any further. If it does not meet the criterion, it is subdivided again into four blocks, and the test criterion is applied to those blocks. This process is repeated iteratively until each block meets the criterion. The result might have blocks of several different sizes.

The function(qtdecomp) first divides the image into four 256-by-256 blocks and then applies the test criterion to each block. If a block does not meet the criterion, qtdecomp subdivides it and applies the test criterion to each block. qtdecomp continues to subdivide blocks until all blocks meet the criterion. Blocks can be as small as 1-by-1, unless you specify, otherwise qtdecomp returns S as a sparse matrix, the same size as I. The nonzero elements of S represent the upper left corners of the blocks; the value of each nonzero element indicates the block size.

qtdecomp divides a square image into four equal-sized square blocks, and then tests each block to see if it meets some criterion of homogeneity. If a block meets the criterion, it is not divided any further. If it does not meet the criterion, it is subdivided again into four blocks, and the test criterion is applied to those blocks. This process is repeated iteratively until each block meets the criterion. The result can have blocks of several different sizes.

qtdecomp is appropriate primarily for square images whose dimensions are a power of 2, such as 128-by-128 or 512-by-512. These images can be divided until the blocks are as small as 1-by-1. If you use qtdecomp with an image whose dimensions are not a power of 2, at some point the blocks cannot be divided further. For example, if an image is 96-by-96, it can be divided into blocks of size 48-by-48, then 24-by-24, 12-by-12, 6-by-6, and finally 3-by-3. No further division beyond 3-by-3 is possible. To process this image, you must set the minimum dimension to 3 (or to 3 times a power of 2); if you are using the syntax that includes a function, the function must return 0 at the point when the block cannot be divided further [(10)] .

6. Matrix Eigenvalues

Eigenvalues are often introduced in the context of linear algebra or matrix theory. Historically, however, they arose in the study of quadratic forms and differential equations. Euler studied the rotational motion of a rigid body and discovered the importance of the principal axes. Lagrange realized that the principal axes are the eigenvectors of the inertia matrix [(11)] .In the early 19th century, Cauchy saw how their work could be used to classify the quadric surfaces, and generalized it to arbitrary dimensions. Cauchy also coined the term *racine caractéristique* (characteristic root) for what is now called *eigenvalue*; his term survives in *characteristic_equation*[(12)] .

The first numerical algorithm for computing eigenvalues and eigenvectors appeared in 1929, when Von Mises published the power method. One of the most popular methods today, the

QR algorithm, was proposed independently by John G.F. Francis and Vera Kublanovskay in 1961[4].

In mathematics, **eigenvalue**, **eigenvector**, and **eigenspace** are related concepts in the field of linear algebra. Linear algebra studies linear transformations, which are represented by matrices acting on vectors.

In general, a matrix acts on a vector by changing both its magnitude and its direction. However, a matrix may act on certain vectors by changing only their magnitude, and leaving their direction unchanged (or, possibly, reversing it). These vectors are the eigenvectors of the matrix[13] .A matrix acts on an eigenvector by multiplying its magnitude by a factor, which is positive if its direction is unchanged and negative if its direction is reversed. This factor is the eigenvalue associated with that eigenvector. An eigenspace is the set of all eigenvectors that have the same eigenvalue. The concepts cannot be formally defined without prerequisites, including an understanding of matrices, vectors, and linear transformations. The technical details are given below.

When a transformation is represented by a square matrix *A*, the eigenvalue equation can be expressed as shown in eq.(1)

$$A\mathbf{x} - \lambda I\mathbf{x} = \mathbf{0}. \qquad \text{……….(1)}$$

This can be rearranged to be shown in eq.(2)

$$(A - \lambda I)\mathbf{x} = \mathbf{0}. \qquad \text{..........(2)}$$

If there exists an inverse to be shown in eq.(3)

$$(A - \lambda I)^{-1}, \qquad \text{..............(3)}$$

then both sides can be left multiplied by the inverse to obtain the trivial solution: **x** = **0**. Thus we require there to be no inverse by assuming from linear algebra that the determinant equals zero:

$$\det(A - \lambda I) = 0.$$

The determinant requirement is called the *characteristic equation* (less often, secular equation) of *A*, and the left-hand side is called the *characteristic polynomial*. When expanded, this gives a polynomial equation for λ. The eigenvector **x** or its components are not present in the characteristic equation(13) .

7. Threshold

Thresholding is the simplest method of image segmentation. From a grayscale image, threshold can be used to create binary images .During the threshold process, individual pixels in an image are marked as “object” pixels if their value is greater than some threshold value (assuming an object to be brighter than the background) and as “background” pixels otherwise. This convention is known as *threshold above*.

Variants include *threshold below*, which is the opposite of threshold above; *threshold inside*, where a pixel is labeled "object" if its value is between two thresholds; and *threshold outside*, which is the opposite of threshold inside. Typically, an object pixel is given a value of “1” while a background pixel is given a value of “0.” Finally, a binary image is created by coloring each pixel white or black, depending on a pixel's label[(14)] .

8. Suggested algorithm:

In this paper a gray image was imported to the system (via flatbed scanner) and then the image is resized into a square image by applying the proposed algorithm continue by the following steps:

1. read a gray image .
2. divide an image into four portions then calculate eigenvalues of each portion and store them in the 4 vectors.

   The function which is used to calculate eigenvalue is (eig) that uses a matrix of type (double) and gives a vector of size (1*128). We apply the first example (natural image) on this function, and the following is a sample of the results.

Table(1) eigenvalue for natural image

<table>
<tr><th colspan="10">Eigenvalue</th></tr>
<tr><th>real</th><th>Imaj</th><th>real</th><th>imaj</th><th>Real</th><th>imaj</th><th>real</th><th>imaj</th><th>real</th><th>imaj</th></tr>
<tr><td colspan="2">1.0e*+004</td><td colspan="2">3.7214+0</td><td colspan="2">-0.3057 +0</td><td colspan="2">-0.1845+0</td><td colspan="2">0.1650</td></tr>
<tr><td colspan="2">0.1187+ 0.0067i</td><td colspan="2">0.1187- 0.0067i</td><td colspan="2">0.0502 + 0.0732i</td><td colspan="2">0.0502 - 0.0732i</td><td colspan="2">+0.0519 - 0.0538i</td></tr>
<tr><td colspan="2">-0.0519 – 0.0538i</td><td colspan="2">0.0611- 0.248i</td><td colspan="2">-0.0611 - 0.248i</td><td colspan="2">0.0361+0.0 434i</td><td colspan="2">-0.0361- 0.0434i</td></tr>
<tr><td colspan="2">-0.0124 – 0.0016i</td><td colspan="2">0.0456</td><td colspan="2">0.0240 - 0.0417i</td><td colspan="2">0.0240 - 0.0417i</td><td colspan="2">0.0424 + 0.0111i</td></tr>
</table>

3. Calculate the mean of the above 4 vectors to get four elements .
4. find the minimums from the previous step, this value is adopted as a threshold value for the segmentation instruction.
5. apply quadtree segmentation process on the gray image, and will produce a segmented image and sparse matrix, using function (qtdecomp) and the instruction is:
   s=qtdecomp(kkk,thru),were the matrix (kkk) is the original image and (thru) is thrushold value which get it from step 4.
6. apply convolution between smooth mask and segmented image to make the segmented image free of sharpness and more smooth .

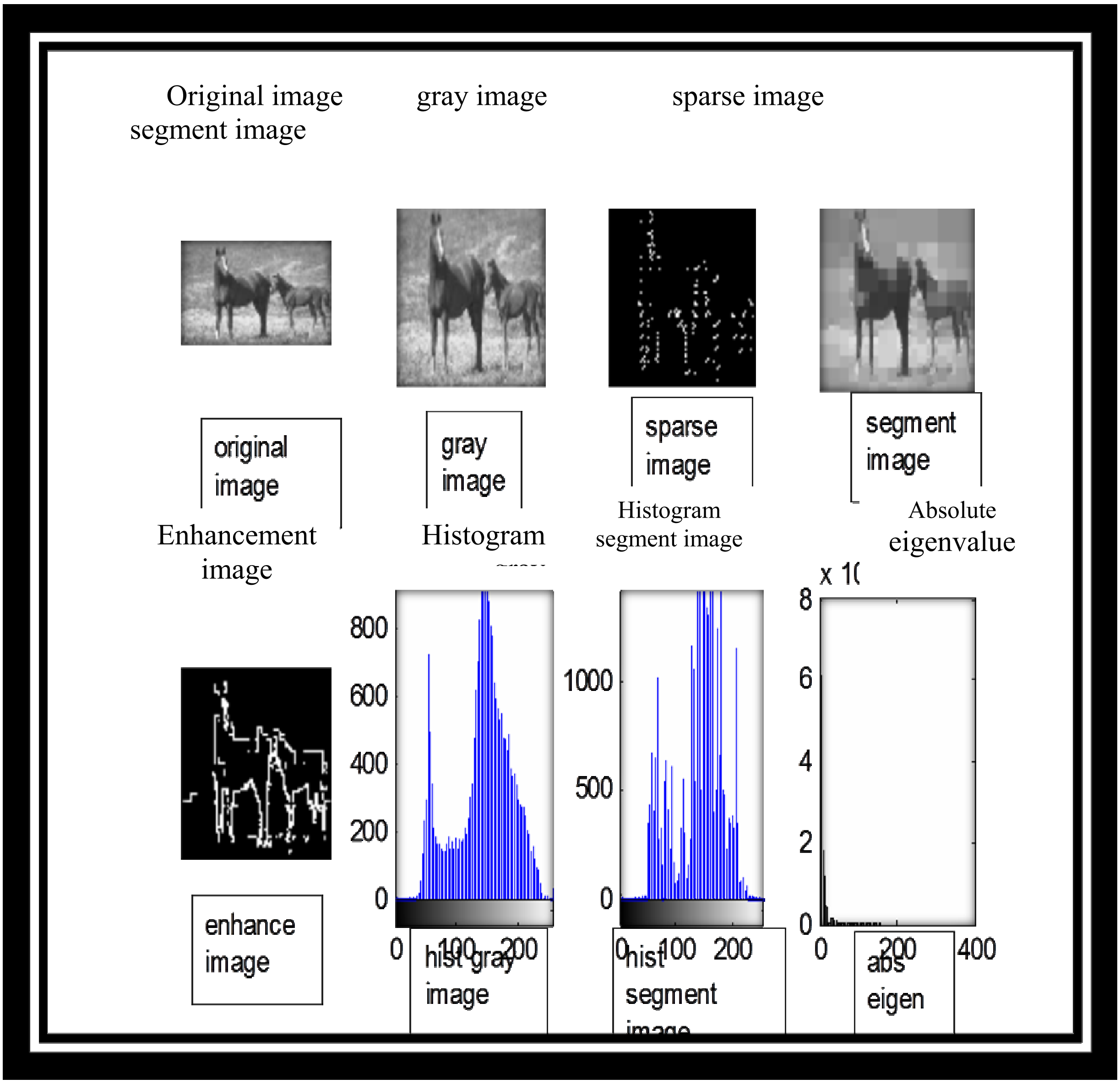
7. calculate absolute value of the Eigen vectors

$$[\text{Eigen}] = \sum_{n=1}^{n} sqrt(\text{Re}^2 + \text{Im}^2)$$

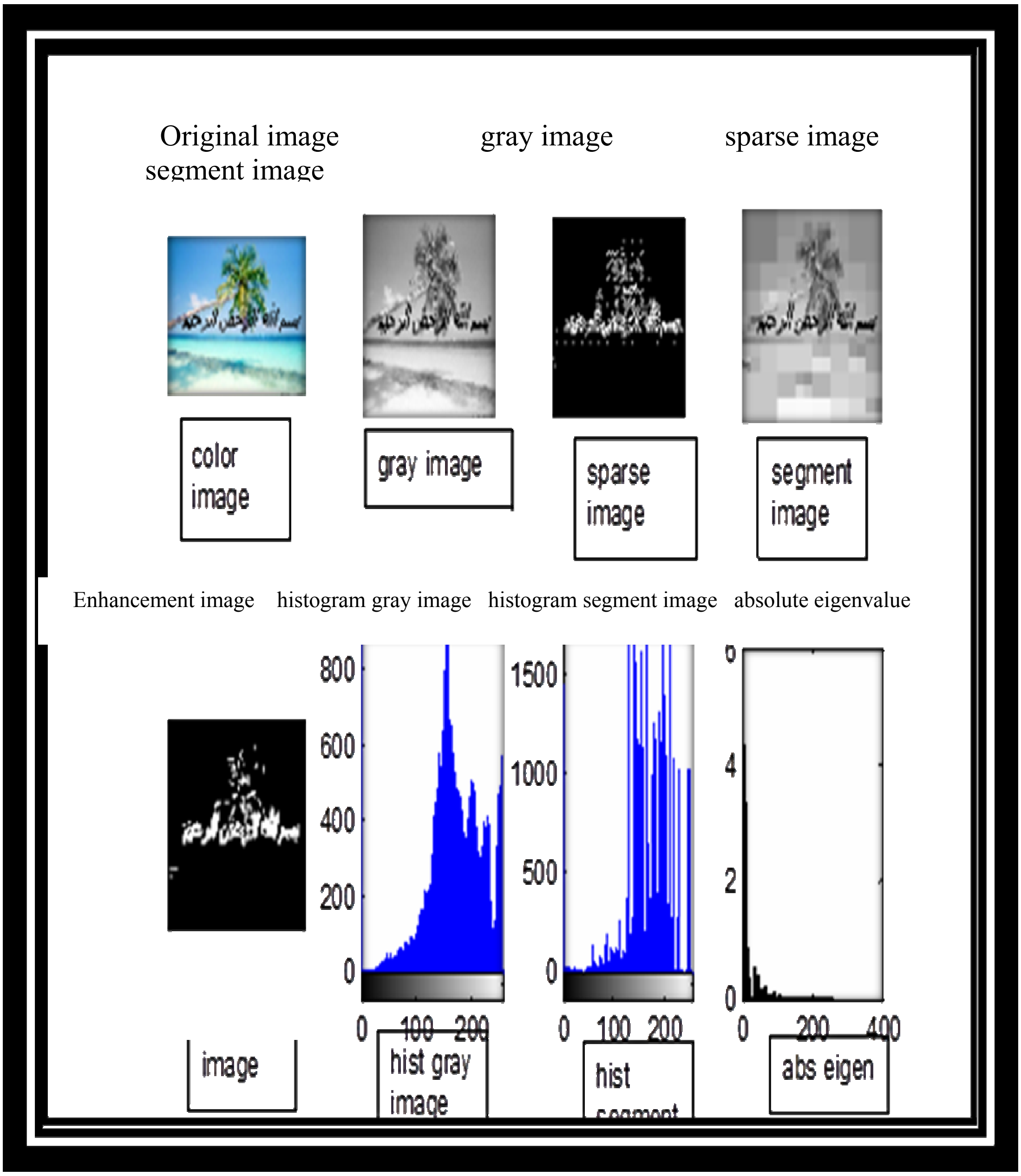
8. plot all results (original image, gray image, sparse matrix, segment image, enhance image, histogram gray image, histogram segment image, absolute eigenvalue).

The first example is a natural image which can be seen in figure(2),we find the eigen values for the original image after segmentation the image in to four parts , for each part the eigen value is calculated ,and calculate the mean for each part then selecting the minimum value (which equall to 114)because it gives best segmentation .the sparse image represent the results of quadtree process which created by connect the pixels image with the result of quadtree,which show the ability of segmentation in this image. Also figure(2) show the segmented image which produced from the quadtree instruction. The using of lowpass filter on the segment image to remove the sharpness and noisy, to show the difference before and after segmentation process the histogram for original gray image and segment image was calculated.

9.Results: 9.RESULTS:



Figure(2) natural image



Figure(3) text image

Figure(3),we find the eigen values for the original image after segmenting the image into four parts , for each part the eigen value is calculated ,and then calculate the mean for each part then selecting the minimum value (which is equal to 148)because it gives best segmentation .the sparse image represents the results of quadtree process which created by connect the pixels image with the result of quadtree,which show the ability of segmentation in this image. Also figure(3) shows the segmented image which produced from the quadtree instruction. The use of lowpass filter on the segment image to remove the sharpness and noisy, to show the difference before and after segmentation process the histogram for original gray image and segment image was calculated.

10.Conclusions

There are many results in this paper:

1. Minimum value of the eigenvalue extracts given best result for segment region, because the minimum value goes to the deep information inside the segmented region.
2. Quadtrees can be manipulated and accessed much quicker than other models. For this reason, quadtrees are very popular in fractal graphics.
3. When eigenvalue is used for image segmentation and applied on different types of images will give an

accurate result for deep information, because it depends on mathematical calculation.

4. Applying quadtree with the proposed algorithm, will make the image manipulation a very powerful process due to the time needed for extracting the property.
5. The inherent recursive nature of quadtrees turns a normally linear data structure into a recursive data structure.
6. Also, the algorithm shows that images represented using quadtrees are more dynamic, for this reason they are well suited for image (imaging in geography and fractals) manipulation.

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