



# Improved Mixed Estimator Using Two Auxiliary Variables For Full Extreme Maximum And Minimum Values In Single Phase Sampling

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## Abstract

The use of multiple auxiliary variables has been established to improve precision in the estimators of ratio, regression and product respectively. However, the presence of extreme values in the distribution could annul such efficiency Olatayo *et al.* (2020). Extreme values could be small or minimum, large or maximum values. This study had developed a ratio-cum-regression estimator with two auxiliary variables, correlation coefficient and coefficient of variation under two types of extreme values in the distribution. This study considers full extreme value cases which assumed that both the study and two auxiliary variables had extreme values present in their distributions. Theoretical, empirical and percentage relative efficiency analyses were carried out for Full High and Maximum Extreme Values (FHMaEV) and Full Low and Minimum Extreme Values cases (FLMiEV). The analysis showed that the developed estimator is efficient over the reviewed estimators.

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## 1. Introduction

The presence of extreme value in the distribution will lead to either over- estimation or under -estimation of the corresponding population parameter Ogunyinka *et al.* (2021). Consequently, this could result in either wrong decision been taken or drawing of invalid inference. The introduction of auxiliary variables in the estimation process of population parameters have been confirmed to improved precision, efficiency and reliability of estimators Ogunyinka *et al.* (2019). A high correlation coefficient between the study variable and the auxiliary variable is an essential factor that is crucial in maximizing the advantages of auxiliary variables Ogunyinka *et al.* (2019). When the correlation between the study variable and the auxiliary variable is positively high and the line of linear regression between the two passes through the origin, then the ratio estimator shows the highest efficiency; but if the line of linear regression does not pass through the origin, the regression estimator has the highest efficiency. When they are highly negative correlated, the product estimator has the highest efficiency; when the variables show a weak correlation only, then the sample mean is preferred (Agunbiade and Ogunyinka, 2013). According to Ogunyinka *et al.* (2019); Cochran (1940) developed the use of ratio estimator in single phase sampling, Raj (1965) introduced the use of more than one auxiliary variable while Mohanty (1967) advanced the use of two estimators in single phase sampling later known as mixed estimation method in in sampling. Ratio-com-regression is an example of mixed estimator and has been confirmed to improve the efficiency of any estimator Ogunyinka *et al.* (2019). The proposed estimator in this study is a combined ratio and regression estimator with full extreme values. The developed estimator used the method developed by Särndal (1972). The combined estimators are the improved ratio and regression estimators of Khan and Shabbir (2013). They were combined in the order of Kadilar and Cingi (2005) while following the procedure of Al-Hossain and Khan (2014). This proposed estimator used one study variable and two auxiliary variables. It assumes that both the study variable and the auxiliary variables have extreme values their distributions and that the

population information of all the auxiliary variables were available. This study has made theoretical and empirical comparison of the proposed estimators with the reviewed estimators of Khan and Shabbir (2013) ratio estimator, Al-Hossain and Khan (2014) ratio estimator and Al-Hossain and Khan (2014) regression estimator in single-phase sampling without replacement. The efficiency of the proposed estimators has been established using the variance, the Mean Square Errors (MSEs). Similarly, the biases of the proposed estimators were ascertained in the empirical analysis. Finally, this study also made use of percentage relative efficiency analysis in order to ascertain by what percentage the proposed estimator is efficient over the reviewed.

## 2. METHODOLOGY

### 2.1. Review on Särndal's Solution to Extreme Values

Särndal (1972) has developed solution to extreme value by introducing a correction constant  $c$  such that if there exists extreme large value in a distribution and  $\bar{y}_{max}$  is the sample mean using Simple Random Sampling without Replacement (SRSWOR), then  $c$  will be subtracted from  $\bar{y}_{max}$  to obtain the corrected mean. This is stated as

$$\bar{y}_1 = \bar{y}_{max} - c \quad (1)$$

Likewise, if there exists extreme low value in a distribution and  $\bar{y}_{min}$  is the sample mean with SRSWOR, then  $c$  will be added to  $\bar{y}_{min}$  to obtain the corrected mean. This is stated as

$$\bar{y}_1 = \bar{y}_{min} + c \quad (2)$$

This can be written in a compressed form as

$$\bar{y}_1 = \begin{cases} \bar{y} + c & \text{if samples contains } y_{min} \text{ but not } y_{max} \\ \bar{y} - c & \text{if samples contains } y_{max} \text{ but not } y_{min} \\ \bar{y} & \text{for all other samples} \end{cases} \quad (3)$$

$c$  is the correction constant. The minimum variance of  $\bar{y}_1$  up to first order of approximation is given as

$$Var(\bar{y}_1)_{min} = \frac{\lambda \Delta^2 y}{2(N-1)} \quad (4)$$

where  $\Delta_y = (y_{max} - y_{min})$  and  $\Delta x = (x_{max} - x_{min})$ . The optimum value of  $c$  is given as

$$c_{opt} = \frac{\Delta y}{2(N-1)} \quad (5)$$

### 2.2. Regression Estimator Using One Auxiliary Variable With Extreme Value

Khan and Shabbir (2013) has proposed an improved regression estimator using one auxiliary variable with extreme value. The estimator is given as

$$\bar{y}_3 = \bar{y}_{c_{11}} + b(\bar{X} - \bar{x}_{c_{21}}) \quad (6)$$

with the corresponding variance as

$$V(\bar{y}_3)_{opt} \cong M(\bar{y}_{lr}) - \frac{\lambda(\Delta y - \beta \Delta x)^2}{2(N-1)} \quad (7)$$

where  $M(\bar{y}_{lr}) = \lambda S_y^2 (1 - \rho_{yx}^2)$  and  $b$  is the sample regression coefficient.

### 2.3. Ratio Estimator Using One Auxiliary Variable With Extreme Value

Khan and Shabbir (2013) has proposed an improved ratio estimator using one auxiliary variable with extreme value. The estimator is given as

$$\bar{y}_4 = \frac{\bar{y}_{c_{11}}}{\bar{x}_{c_{21}}} \bar{X}. \quad (8)$$

The corresponding MSE is given as:

$$MSE(\bar{y}_4)_{opt} \cong M(\bar{y}_R) - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)}, \quad (9)$$

where  $M(\bar{y}_R) \cong \bar{Y}^2 \lambda(C_y^2 C_x^2 - 2\rho_{yx} C_y C_x)$ , is the mean square error of the conventional ratio estimator.

### 2.4. Ratio Estimator Using Two Auxiliary Variables With Extreme Value

Al-Hossain and Khan (2014) has proposed an improved ratio estimator using two auxiliary variables with extreme value. The estimator is given as

$$\bar{y}_5 = \bar{y}_{c_{11}} \left( \frac{\bar{X}_1}{\bar{x}_{1c_{21}}} \right) \left( \frac{\bar{X}_2}{\bar{x}_{2c_{31}}} \right). \quad (10)$$

The corresponding MSE is presented as

$$MSE(\bar{y}_5)_{opt} \cong M(\bar{y}_{R2}) - \frac{\lambda(\Delta y - R_1 \Delta x_1 - R_2 \Delta x_2)^2}{2(N-1)}, \quad (11)$$

where  $M(\bar{y}_{R2}) = \lambda(S_y^2 + R_1^2 S_{x_1}^2 + R_2^2 S_{x_2}^2 + 2R_1 R_2 S_{x_1 x_2} - 2R_2 S_{y x_2} - 2R_1 S_{y x_1})$ .

### 2.5. Regression Estimator Using Two Auxiliary Variables With Extreme Value

Al-Hossain and Khan (2014) has proposed an improved regression estimator using two auxiliary variables with extreme value. The improved regression estimator of Al-Hossain and Khan (2014) is given as

$$\bar{y}_6 = \bar{y}_{c_{11}} + b_1 (\bar{X} - \bar{x}_{1c_{21}}) + b_2 (\bar{X} - \bar{x}_{2c_{31}}). \quad (12)$$

The corresponding MSE given as

$$MSE(\bar{y}_6)_{opt} \cong M(\bar{y}_{lr}) - \frac{\lambda(\Delta y - \beta_1 \Delta x_1 - \beta_2 \Delta x_2)^2}{2(N-1)} \quad (13)$$

where  $M(\bar{y}_{lr}) = \lambda S_y^2 (1 - \rho_{y x_1}^2 - \rho_{y x_2}^2 + 2\rho_{y x_1} \rho_{y x_2} \rho_{x_1 x_2})$ . Similarly,

$\beta_1 = \rho_{y x_1} \frac{S_y}{S_{x_1}}$  and  $\beta_2 = \rho_{y x_2} \frac{S_y}{S_{x_2}}$  are the population regression coefficient between  $y$  and  $x_1$  and between  $y$  and  $x_2$ .

### 3. Proposed Estimator

This study has proposed an improved mixed estimator derived by combining the improved ratio and regression estimators of Khan and Shabbir (2013) in single phase sampling without replacement. The proposed estimator has been termed Full Extreme Value in both Study and Auxiliary variable (FEVSA) and is denoted by:

$$\bar{y}_7 = \left( \frac{\bar{y}_{c_1}}{\bar{x}_{c_{11}}} \right) \bar{X}_1 + b(\bar{X}_2 - \bar{x}_{c_{22}}) \quad (14)$$

The relative error terms are defined as

$$\left. \begin{aligned} \bar{y}_{c_1} &= \bar{Y}(1 + \varepsilon_0) \\ \bar{x}_{c_{11}} &= \bar{X}_1(1 + \varepsilon_1) \\ \bar{x}_{c_{22}} &= \bar{X}_2(1 + \varepsilon_2) \end{aligned} \right\} \quad (15)$$

such that

$$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$$

The estimators  $\bar{y}_{c_1} = (\bar{y} + c_1)$ ,  $\bar{x}_{c_{11}} = (\bar{x}_1 + c_1)$ , and  $\bar{x}_{c_{22}} = (\bar{x}_2 + c_2)$  provided the sample contains  $y_{min}$ ,  $x_{1min}$  and  $x_{2min}$ . Similarly, the estimators  $\bar{y}_{c_1} = (\bar{y} - c_1)$ ,  $\bar{x}_{c_{11}} = (\bar{x}_1 - c_1)$ ,  $\bar{x}_{c_{22}} = (\bar{x}_2 - c_2)$ . If the sample contains  $y_{max}$ ,  $x_{1max}$

and  $x_{2max}$ . Finally, the estimator  $\bar{y}_{c_1} = \bar{y}$ ,  $\bar{x}_{c_{11}} = \bar{x}_1$ ,  $\bar{x}_{c_{22}} = x_2$  for all other combinations of samples. The following expectation would be used.

$$\left. \begin{aligned} E(\varepsilon_0^2) &= \frac{\lambda}{\bar{Y}^2} \left[ s_y^2 - \frac{2nc_0}{N-1} (\Delta y - nc_0) \right] \\ E(\varepsilon_1^2) &= \frac{\lambda}{\bar{X}_1^2} \left[ s_{x_1}^2 - \frac{2nc_1}{N-1} (\Delta x_1 - nc_1) \right] \\ E(\varepsilon_2^2) &= \frac{\lambda}{\bar{X}_2^2} \left[ s_{x_2}^2 - \frac{2nc_2}{N-1} (\Delta x_2 - nc_2) \right] \end{aligned} \right\} \quad (16)$$

and

$$\left. \begin{aligned} E(\varepsilon_0 \varepsilon_1) &= \frac{\lambda}{\bar{Y} \bar{X}_1} \left[ s_{yx_1} - \frac{n}{N-1} (c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1) \right] \\ E(\varepsilon_0 \varepsilon_2) &= \frac{\lambda}{\bar{Y} \bar{X}_2} \left[ s_{yx_2} - \frac{n}{N-1} (c_2 \Delta y + c_0 \Delta x_2 - 2nc_0 c_2) \right] \\ E(\varepsilon_1 \varepsilon_2) &= \frac{\lambda}{\bar{X}_1 \bar{X}_2} \left[ s_{x_1 x_2} - \frac{n}{N-1} (c_2 \Delta x_1 + c_1 \Delta x_2 - 2nc_1 c_2) \right] \end{aligned} \right\}. \quad (17)$$

Substituting equation (15) into equation (14) will give

$$\bar{y}_8 = \bar{Y}(1 + \varepsilon_0)(1 + \varepsilon_1)^{-1} - b\varepsilon_2 \bar{X}_2$$

Applying Taylor series of expanding  $(1 + \varepsilon_1)^{-1}$  up to second order of degree, will give

$$\begin{aligned} \bar{y}_7 &= \bar{Y}(1 - \varepsilon_1 + \varepsilon_1^2 + \varepsilon_0 - \varepsilon_0 \varepsilon_1) - b\varepsilon_2 \bar{X}_2 \\ \text{Bias}(\bar{y}_7) &= E(\bar{y}_8 - \bar{Y}) \end{aligned} \quad (18)$$

but

$$\bar{y}_7 - \bar{Y} = \bar{Y}(1 - \varepsilon_1 + \varepsilon_1^2 + \varepsilon_0 - \varepsilon_0 \varepsilon_1) - b\varepsilon_2 \bar{X}_2 - \bar{Y}$$

$$\text{Bias}(\bar{y}_7) = E[\bar{Y}(1 - \varepsilon_1 + \varepsilon_1^2 + \varepsilon_0 - \varepsilon_0 \varepsilon_1) - b\varepsilon_2 \bar{X}_2]$$

Applying expectation,

$$\text{Bias}(\bar{y}_7) = \bar{Y}[E(\varepsilon_1^2) - E(\varepsilon_0 \varepsilon_1)]$$

Substituting and collecting some like terms, gives

$$\begin{aligned} \text{Bias}(\bar{y}_7) &= \frac{\bar{Y}\lambda}{\bar{X}_1^2} \left[ s_{x_1}^2 - \frac{2nc_1}{N-1} (\Delta x_1 - nc_1) \right] - \frac{\bar{Y}\lambda}{\bar{Y}\bar{X}_1} \left[ s_{yx_1} \frac{n}{N-1} (c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1) \right] \\ \text{Bias}(\bar{y}_7) &= \frac{R_1^2 \lambda}{\bar{Y}} \left[ s_{x_1}^2 - \frac{2nc_1}{N-1} (\Delta x_1 - nc_1) \right] - \frac{R_1 \lambda}{\bar{Y}} \left[ s_{yx_1} \frac{n}{N-1} (c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1) \right] \\ &= \frac{R_1 \lambda}{\bar{Y}} \left[ (R_1 s_{x_1}^2 - s_{yx_1}) \right] - \frac{R_1^2 \lambda}{\bar{Y}} \left[ \frac{2nc_1 (\Delta x_1 - nc_1)}{N-1} \right] - \frac{R_1 \lambda n}{\bar{Y}(N-1)} [(c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1)] \\ \text{Bias}(\bar{y}_7) &= \frac{R_1 \lambda}{\bar{Y}} \left\{ [(R_1 s_{x_1}^2 - s_{yx_1})] - \frac{n}{(N-1)} [(2c_1 R_1 (\Delta x_1 - nc_1)) \right. \\ &\quad \left. + (c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1)] \right\} \end{aligned} \quad (19)$$

Substituting the optimum values of  $c_0$ ,  $c_1$  and  $c_2$  into (3.30), where

$$c_0 = \frac{\Delta y}{2n}, \quad c_1 = \frac{\Delta x_1}{2n}, \quad c_2 = \frac{\Delta x_2}{2n}, \text{ gives}$$

$$\text{Bias}(\bar{y}_7) = \frac{R_1 \lambda}{\bar{Y}} \left[ (R_1 s_{x_1}^2 - s_{yx_1}) - \frac{1}{2(N-1)} (R_1 \Delta^2 x_1 + \Delta x_1 \Delta y) \right] \quad (20)$$

$$\text{Similarly, } \text{MSE}(\bar{y}_7) = E(\bar{y}_8 - \bar{Y})^2 \quad (21)$$

$$= E[\bar{Y}(1 + \varepsilon_0)(1 + \varepsilon_1)^{-1} - b\varepsilon_2 \bar{X}_2]^2 \quad (22)$$

Applying expectation after expanding (20) gives

$$\begin{aligned} \text{MSE}(\bar{y}_7) &= \bar{Y}^2 \left\{ \frac{\lambda}{\bar{Y}^2} \left[ s_y^2 - \frac{2nc_0}{N-1} (\Delta y - nc_0) \right] \right\} + \bar{Y}^2 \left\{ \frac{\lambda}{\bar{X}_1^2} \left[ s_{x_1}^2 - \frac{2nc_1}{N-1} (\Delta x_1 - nc_1) \right] \right\} \\ &\quad - 2\bar{Y}^2 \left\{ \frac{\lambda}{\bar{Y}\bar{X}_1} \left[ s_{yx_1} - \frac{n}{N-1} (c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1) \right] \right\} + b^2 \bar{X}_2^2 \left\{ \frac{\lambda}{\bar{X}_2^2} \left[ s_{x_2}^2 - \frac{2nc_2}{N-1} (\Delta x_2 - nc_2) \right] \right\} \\ &\quad - 2b\bar{X}_2 \bar{Y} \left[ s_{yx_2} - \frac{n}{N-1} (c_2 \Delta y + c_0 \Delta x_2 - 2nc_0 c_2) \right] \\ &\quad + 2b\bar{X}_2 \bar{Y} \left\{ \frac{\lambda}{\bar{X}_2 \bar{Y}} \left[ s_{x_1 x_2} - \frac{n}{N-1} (c_2 \Delta x_1 + c_1 \Delta x_2 - 2nc_1 c_2) \right] \right\} \end{aligned}$$

$$\begin{aligned}
 MSE(\bar{y}_7) &= [\lambda s_y^2 + \lambda R_1^2 s_{x_1}^2 - 2R_1 \lambda s_{yx_1} + b^2 \lambda s_{x_2}^2 - 2b \lambda s_{yx_2} + 2b R_1 \lambda s_{x_1 x_2}] \\
 &\quad - \left\{ \lambda \left[ \frac{2nc_0}{N-1} (\Delta y - nc_0) \right] + \lambda R_1^2 \left[ -\frac{2nc_1}{N-1} (\Delta x_1 - nc_1) \right] - 2R_1 \lambda \left[ -\frac{n}{N-1} (c_1 \Delta y + c_0 \Delta x_1 - 2nc_0 c_1) \right] \right. \\
 &\quad + b^2 \lambda \left[ -\frac{2nc_2}{N-1} (\Delta x_2 - nc_2) \right] - 2b \lambda \left[ -\frac{n}{N-1} (c_2 \Delta y + c_0 \Delta x_2 - 2nc_0 c_2) \right] \\
 &\quad \left. + 2b R_1 \lambda \left[ -\frac{n}{N-1} (c_2 \Delta x_1 + c_1 \Delta x_2 - 2nc_1 c_2) \right] \right\} \\
 MSE(\bar{y}_7) &= \lambda [s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2b R_1 s_{x_1 x_2}] \\
 &\quad - \frac{2\lambda n}{N-1} [\Delta y c_0 - c_0^2 n + R_1^2 c_1 \Delta x_1 - R_1^2 c_1^2 n - R_1 c_1 \Delta y - R_1 c_0 \Delta x_1 + 2R_1 c_0 c_1 n + b^2 c_2 \Delta x_2 - b^2 c_2^2 n \\
 &\quad - b c_2 \Delta y - b c_0 \Delta x_2 + 2b n c_0 c_2 + b R_1 c_2 \Delta x_1 + b R_1 c_1 \Delta x_2 \\
 &\quad - 2b R_1 c_1 c_2 n] \tag{23}
 \end{aligned}$$

To obtain,  $\hat{b}_{opt}$  differentiating (21) and equating to zero

$$\begin{aligned}
 \frac{\partial(M(\bar{y}_8))}{\partial(b)} &= 2\lambda b s_{x_2}^2 - 2\lambda s_{yx_2} + 2\lambda R_1 s_{x_1 x_2} \\
 &\quad - \frac{2\lambda n}{N-1} \{2b c_2 \Delta x_2 - 2b c_2^2 n - c_2 \Delta y - c_0 \Delta x_2 + 2n c_0 c_2 + R_1 c_2 \Delta x_1 + R_1 c_1 \Delta x_2 - 2R_1 c_1 c_2 n\} = 0 \\
 \Rightarrow 2\lambda [b s_{x_2}^2 - s_{yx_2} + R_1 s_{x_1 x_2}] &= \frac{2\lambda n}{N-1} \{2b c_2 \Delta x_2 - 2b c_2^2 n - c_2 \Delta y - c_0 \Delta x_2 + 2n c_0 c_2 + R_1 c_2 \Delta x_1 + R_1 c_1 \Delta x_2 - 2R_1 c_1 c_2 n\} \\
 \Rightarrow b s_{x_2}^2 - \frac{b n}{N-1} (2c_2 \Delta x_2 - 2c_2^2 n) \\
 &= s_{yx_2} - R_1 s_{x_1 x_2} + \frac{n}{N-1} [-c_2 \Delta y - c_0 \Delta x_2 + 2n c_0 c_2] + \frac{n}{N-1} [R_1 c_2 \Delta x_1 + R_1 c_1 \Delta x_2 - 2R_1 c_1 c_2 n]
 \end{aligned}$$

Hence,

$$\hat{b} = \frac{s_{yx_2} - R_1 s_{x_1 x_2} + \frac{n}{N-1} [-c_2 \Delta y - c_0 \Delta x_2 + 2n c_0 c_2 + R_1 c_2 \Delta x_1 + R_1 c_1 \Delta x_2 - 2R_1 c_1 c_2 n]}{s_{x_2}^2 - \frac{n}{N-1} [2c_2 \Delta x_2 - 2c_2^2 n]} \tag{24}$$

To obtain the optimum values of  $b$ , substituting  $c_0, c_1, c_2$  into equation (22), it gives

$$\begin{aligned}
 \hat{b}_{opt} &= \frac{2[s_{yx_2} - R_1 s_{x_1 x_2}](N-1) + \Delta x_2 [R_1 \Delta x_1 - \Delta y]}{2s_{x_2}^2 (N-1) - \Delta^2 x_2} \quad \text{or } \hat{b}_{opt} = \\
 &= \frac{[s_{yx_2} - R_1 s_{x_1 x_2}](N-1) + \frac{\Delta x_2}{2} [R_1 \Delta x_1 - \Delta y]}{s_{x_2}^2 (N-1) - \frac{\Delta^2 x_2}{2}} \tag{25}
 \end{aligned}$$

The  $MSE(\bar{y}_8)_{min}$  is obtained by substituting the optimum values of  $c_0, c_1, c_2$  and  $\hat{b}_{opt}$  into (21) as follows:

First, substitute the optimum values of  $c_0, c_1, c_2$  into equation (21), it gives

$$\begin{aligned}
 MSE(\bar{y}_8)_{min} &= \lambda [s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2b R_1 s_{x_1 x_2}] \\
 &\quad - \frac{2\lambda n}{N-1} \left[ \frac{\Delta y}{2n} \Delta y - \left( \frac{\Delta y}{2n} \right)^2 n + R_1^2 \left( \frac{\Delta x_1}{2n} \right) \Delta x_1 - R_1^2 \left( \frac{\Delta x_1}{2n} \right)^2 n \right] \\
 &\quad - \frac{2\lambda n}{N-1} \left[ -R_1 \left( \frac{\Delta x_1}{2n} \right) \Delta y - R_1 \left( \frac{\Delta x_1}{2n} \right) \Delta y + 2R_1 \left( \frac{\Delta x_1}{2n} \frac{\Delta y}{2n} \right) n \right] \\
 &\quad - \frac{2\lambda n}{N-1} \left[ +b^2 \left( \frac{\Delta x_2}{2n} \right) \Delta x_2 - b^2 \left( \frac{\Delta x_2}{2n} \right)^2 n - b \left( \frac{\Delta x_2}{2n} \right) \Delta y - b \left( \frac{\Delta x_2}{2n} \right) \Delta y \right] \\
 &\quad - \frac{2\lambda n}{N-1} \left[ 2bn \left( \frac{\Delta x_2}{2n} \frac{\Delta y}{2n} \right) + b R_1 \left( \frac{\Delta x_2}{2n} \right) \Delta x_1 + b R_1 \left( \frac{\Delta x_1}{2n} \right) \Delta x_2 - 2bn R_1 \left( \frac{\Delta x_1}{2n} \frac{\Delta x_2}{2n} \right) \right] \\
 MSE(\bar{y}_7)_{min} &= \lambda [s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2b R_1 s_{x_1 x_2}] \\
 &\quad - \frac{2\lambda n}{N-1} \left[ \frac{\Delta^2 y}{4n} + \frac{R_1^2 \Delta^2 x_1}{4n} - \frac{R_1 \Delta x_1 \Delta y}{2n} + \frac{b^2 \Delta_x^2}{4n} - \frac{b \Delta x_2 \Delta y}{2n} + \frac{b R_1 \Delta x_2 \Delta x_1}{2n} \right] \\
 &\quad - \frac{\lambda}{N-1} \left[ \frac{\Delta^2 y}{2} + \frac{R^2 \Delta^2 x_1}{2} - R_1 \Delta x_1 \Delta y + b^2 \Delta^2 x_2 - b \Delta x_2 \Delta y + b R_1 \Delta x_2 \Delta x_1 \right]
 \end{aligned}$$

$$\begin{aligned}
 MSE(\bar{y}_7)_{min} = & \lambda [s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2b R_1 s_{x_1 x_2}] \\
 & - \frac{\lambda}{2(N-1)} [\Delta^2 y + R_1^2 \Delta^2 x_1 - 2R_1 \Delta x_1 \Delta y + b^2 \Delta^2 x_2 - 2b \Delta x_2 \Delta y \\
 & + 2b R_1 \Delta x_2 \Delta x_1]
 \end{aligned} \tag{26}$$

Hence,

$$\begin{aligned}
 MSE(\bar{y}_7)_{min} = & \lambda [s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1}] - \frac{\lambda}{2(N-1)} [\Delta y - R_1 \Delta x_1]^2 + \lambda b^2 \left[ \frac{2s_{x_2}^2 (N-1) - \Delta^2 x_2}{2(N-1)} \right] \\
 & + \lambda [2b R_1 s_{x_1 x_2} - 2b s_{yx_2}] - \frac{\lambda}{2(N-1)} [2b R_1 \Delta x_1 \Delta x_2 - 2b \Delta x_2 \Delta y]
 \end{aligned}$$

Next, substituting  $\hat{b}_{opt}$  into equation (24),

$$\begin{aligned}
 MSE(\bar{y}_7)_{min} = & \lambda [s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b(s_{yx_2} - R_1 s_{x_1 x_2})] \\
 & - \frac{\lambda}{2(N-1)} [(\Delta_y^2 + R_1^2 \Delta x_1^2 - 2R_1 \Delta_y \Delta x_1 + b^2 \Delta x_1^2 - 2b(\Delta_y \Delta x_2 \\
 & - R_1 \Delta x_2 \Delta x_1))
 \end{aligned} \tag{27}$$

## Results

### 4.1 Theoretical Comparison Of FEVSA-Case With The Reviewed Estimators.

The condition will be if  $MSE(\bar{y}_1)_{min}$  minus the  $MSE$  of any of the reviewed is less than zero is satisfied, then  $\bar{y}_1$  is more efficient, otherwise, decision will be reverse.

#### A. Comparing Fevsa-Case With Mse Of Khan And Shabbir (2013) Improved Ratio Estimator

$$\begin{aligned}
 & MSE(\bar{y}_7)_{min} - MSE(\bar{y}_4)_{min} < 0 \\
 & \lambda[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b(s_{yx_2} - R_1 s_{x_1 x_2})] \\
 & - \frac{\lambda}{2(N-1)} [(\Delta_y^2 + R_1^2 \Delta x_1^2 - 2R_1 \Delta_y \Delta x_1 + b^2 \Delta x_2^2 - 2b(R_1 \Delta x_2 \Delta x_1 - \Delta_y \Delta x_2)) - \lambda \left[ S_y^2 + R^2 S_y^2 - 2 \frac{RS_{xy}}{\bar{Y}^2} \right] \\
 & - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)} < 0 \\
 & \lambda[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1}] + \lambda b^2 \left[ s_{x_2}^2 - \frac{\Delta x_2^2}{2(N-1)} \right] + 2b\lambda[R_1 s_{x_1 x_2} - s_{yx_2}] - \frac{\lambda}{2(N-1)} [\Delta_y^2 + R_1^2 \Delta x_1^2 - 2R_1 \Delta_y \Delta x_1] \\
 & - \frac{\lambda}{2(N-1)} [2b(R_1 \Delta x_2 \Delta x_1 - \Delta_y \Delta x_2)] - \left\{ \lambda \left[ s_y^2 + R^2 s_y^2 - 2 \frac{RS_{xy}}{\bar{Y}^2} \right] - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)} \right\} \\
 & < 0 \quad (28)
 \end{aligned}$$

This implies that  $\bar{y}_7$  is more efficient than  $\bar{y}_4$

#### b. Comparing FEV-Case with AL-Hossain and Khan (2014) ratio Estimator

$$MSE(\bar{y}_7)_{min} - MSE(\bar{y}_5)_{min} < 0$$

This implies that

$$\begin{aligned}
 & \lambda[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b(s_{yx_2} - R_1 s_{x_1 x_2})] \\
 & - \frac{\lambda}{2(N-1)} [(\Delta_y^2 + R_1^2 \Delta x_1^2 - 2R_1 \Delta_y \Delta x_1 + b^2 \Delta x_2^2 - 2b(\Delta_y \Delta x_2 - R_1 \Delta x_2 \Delta x_1)) \\
 & - \left\{ \lambda(S_y^2 + R_1^2 S_{x_1}^2 + R_2^2 S_{x_2}^2 + 2R_1 R_2 S_{x_1 x_2} - 2R_2 S_{yx_2} - 2R_1 S_{yx_1}) - \frac{\lambda(\Delta y - R_1 \Delta x_1 - R_2 \Delta x_2)^2}{2(N-1)} \right\}] \\
 & \lambda \left[ b^2 \left( s_{x_2}^2 - \frac{\Delta x_2^2}{2(N-1)} + 2b(R_1 s_{x_1 x_2} - s_{yx_2}) \right) \right] - \lambda \left[ R_2^2 S_{x_2}^2 + 2R_1 R_2 S_{x_1 x_2} - 2R_1 S_{yx_2} \right] - \frac{\lambda}{2(N-1)} \\
 & [2b(R_1 \Delta x_2 \Delta x_1 - \Delta_y \Delta x_2)] - \frac{\lambda}{2(N-1)} [\Delta_y^2 + R_1^2 \Delta x_1^2 - 2R_1 \Delta_y \Delta x_1] + \frac{\lambda(\Delta y - R_1 \Delta x_1 - R_2 \Delta x_2)^2}{2(N-1)} \\
 & < 0 \quad (29)
 \end{aligned}$$

This implies that  $\bar{y}_7$  is superior to  $\bar{y}_5$

#### c. Comparing FEV-Case with AL-Hossain and Khan (2014) of regression estimator

$$MSE(\bar{y}_7)_{min} - MSE(\bar{y}_6)_{min} < 0$$

$$\begin{aligned}
 & \lambda[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b(s_{yx_2} - R_1 s_{x_1 x_2})] \\
 & - \frac{\lambda}{2(N-1)} [(\Delta_y^2 + R_1^2 \Delta x_1^2 - 2R_1 \Delta_y \Delta x_1 + b^2 \Delta x_2^2 - 2b(R_1 \Delta x_2 \Delta x_1 - \Delta_y \Delta x_2))] \\
 & - \left\{ S_y^2 \left[ 1 - b_1^2 \frac{S_{x_1}^2}{S_y^2} - b_2^2 \frac{S_{x_2}^2}{S_y^2} \right] + \lambda S_y^2 \left[ 2b_1 \frac{S_{x_1}}{S_y} b_2 \frac{S_{x_2}}{S_y} \frac{S_{x_1 x_2}}{S_{x_1} S_{x_2}} \right] - \frac{\lambda(\Delta y - b_1 \Delta x_1 - b_2 \Delta x_2)^2}{2(N-1)} \right\} \\
 & < 0 \quad (30)
 \end{aligned}$$

The efficiency of  $\bar{y}_7$  over  $\bar{y}_6$  will be obtained empirically using equation (30).

#### 4. Empirical Analysis

##### 5.1 Comparison of FEVSA with the reviewed estimators for the case of High and Maximum Extreme Values (HMAEV) table 1-3.

**Table1.** MSE comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMAEV

| S/N Populations         | 1        | 2        | 3        | 4        | 5        | 6        | 7        |
|-------------------------|----------|----------|----------|----------|----------|----------|----------|
| 1. MSE ( $\bar{y}_7$ )  | 9039.007 | 3585.255 | 9454.076 | 8186.263 | 7190.929 | 8555.007 | 10067.8  |
| 2. MSE ( $\bar{y}_4$ )  | 253180.2 | 92865.49 | 271747.6 | 226558.7 | 193342.2 | 254878.2 | 313252.8 |
| 3. MSE ( $\bar{y}_5$ )  | 13928.04 | 7307.471 | 16462.41 | 14393.79 | 12197.14 | 16976.95 | 22577.38 |
| 4. MSE ( $\bar{y}_6$ )  | 9572.582 | 4243.1   | 10553.71 | 8893.569 | 7672.536 | 10508.48 | 12693.74 |
| Rank MSE( $\bar{y}_1$ ) | 1        | 1        | 1        | 1        | 1        | 1        | 1        |
| Rank MSE( $\bar{y}_4$ ) | 4        | 4        | 4        | 4        | 4        | 4        | 4        |
| Rank MSE( $\bar{y}_5$ ) | 3        | 3        | 3        | 3        | 3        | 3        | 3        |
| Rank MSE( $\bar{y}_6$ ) | 2        | 2        | 2        | 2        | 2        | 2        | 2        |

**Table 2:** MSE Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMAEV cases (continue)

| S/N Populations        | 8        | 9        | 10       | 11       | 12       | 13       | 14       |
|------------------------|----------|----------|----------|----------|----------|----------|----------|
| 1. MSE ( $\bar{y}_7$ ) | 13153.84 | 21964.61 | 25480.39 | 30147.24 | 38698.05 | 15043.49 | 16986.67 |
| 2. MSE ( $\bar{y}_4$ ) | 419045.3 | 754112   | 923106.7 | 1177701  | 1612307  | 573734.1 | 667053.4 |
| 3. MSE ( $\bar{y}_5$ ) | 27112.14 | 34497.47 | 50521.47 | 62759.85 | 88084.77 | 45497.15 | 50967.38 |

|                                         |                 |                 |                 |                 |                 |                 |                 |
|-----------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| <b>4. MSE (<math>\bar{y}_6</math>)</b>  | <b>15921.55</b> | <b>23275.36</b> | <b>29689.32</b> | <b>38058.76</b> | <b>48689.05</b> | <b>23201.94</b> | <b>26482.01</b> |
| <b>Rank MSE(<math>\bar{y}_7</math>)</b> | <b>1</b>        | <b>1</b>        | <b>1</b>        | <b>1</b>        | <b>1</b>        | <b>1</b>        | <b>1</b>        |
| <b>Rank MSE(<math>\bar{y}_4</math>)</b> | <b>4</b>        | <b>4</b>        | <b>4</b>        | <b>4</b>        | <b>4</b>        | <b>4</b>        | <b>4</b>        |
| <b>Rank MSE(<math>\bar{y}_5</math>)</b> | <b>3</b>        | <b>3</b>        | <b>3</b>        | <b>3</b>        | <b>3</b>        | <b>3</b>        | <b>3</b>        |
| <b>Rank MSE(<math>\bar{y}_6</math>)</b> | <b>2</b>        | <b>2</b>        | <b>2</b>        | <b>2</b>        | <b>2</b>        | <b>2</b>        | <b>2</b>        |

**Table 3:** MSE Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV cases

| S/N | Populations             | 15       | 16       | 17       | 18       | 19       | 20       | Overall Ranking |
|-----|-------------------------|----------|----------|----------|----------|----------|----------|-----------------|
| 1.  | MSE ( $\bar{y}_7$ )     | 104641.2 | 39447.89 | 33902.91 | 111080.3 | 309540.7 | 649428.8 |                 |
| 2.  | MSE ( $\bar{y}_4$ )     | 6258080  | 1956279  | 2026384  | 7303558  | 31577741 | 90611571 |                 |
| 3.  | MSE ( $\bar{y}_5$ )     | 174441.8 | 139203.4 | 184225.9 | 452657.2 | 1432352  | 3191902  |                 |
| 4.  | MSE ( $\bar{y}_6$ )     | 111874.1 | 68002.77 | 79395.13 | 192497   | 629051   | 1290081  |                 |
|     | Rank MSE( $\bar{y}_7$ ) | 1        | 1        | 1        | 1        | 1        | 1        | 1               |
|     | Rank MSE( $\bar{y}_4$ ) | 4        | 4        | 4        | 4        | 4        | 4        | 4               |
|     | Rank MSE( $\bar{y}_5$ ) | 3        | 3        | 3        | 3        | 3        | 3        | 3               |
|     | Rank MSE( $\bar{y}_6$ ) | 2        | 2        | 2        | 2        | 2        | 2        | 2               |

## 5.2 Comparison of FEVSA with the reviewed estimators for the case of Low and Minimum Extreme Values (LMiEV) table 4-6.

Table 4. Comparison of the proposed estimators with the reviewed estimators for the twenty stimulated populations for LMiEV

| S/N                     | 1          | 2           | 3           | 4           | 5          | 6           | 7          |
|-------------------------|------------|-------------|-------------|-------------|------------|-------------|------------|
| <b>Populations</b>      |            |             |             |             |            |             |            |
| 1. MSE ( $\bar{y}_7$ )  | 4639.3836  | 9647.557    | 13363.6194  | 9734.5354   | 13803.6141 | 13382.7563  | 9367.2453  |
| 2. MSE ( $\bar{y}_4$ )  | 62526.0429 | 80547.27837 | 95128.42097 | 91465.53736 | 108745.356 | 94488.16596 | 109258.909 |
| 3. MSE ( $\bar{y}_5$ )  | 7530.87287 | 11565.93007 | 15222.01492 | 12934.9066  | 17750.6313 | 15748.2352  | 14597.4654 |
| 4. MSE ( $\bar{y}_6$ )  | 5095.52272 | 9645.987777 | 13355.4278  | 9921.678177 | 13977.5592 | 13385.63772 | 10373.735  |
| Rank MSE( $\bar{y}_7$ ) | 1          | 2           | 2           | 1           | 1          | 1           | 1          |
| Rank MSE( $\bar{y}_4$ ) | 4          | 4           | 4           | 4           | 4          | 4           | 3          |
| Rank MSE( $\bar{y}_5$ ) | 3          | 3           | 3           | 3           | 3          | 3           | 3          |
| Rank MSE( $\bar{y}_6$ ) | 2          | 1           | 1           | 2           | 2          | 2           | 2          |

**Table 5:** Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for LMiEV cases

| S/N | Populations             | 8          | 9          | 10         | 11         | 12          | 13          | 14         |
|-----|-------------------------|------------|------------|------------|------------|-------------|-------------|------------|
| 1.  | MSE ( $\bar{y}_7$ )     | 28789.6481 | 27995.2403 | 33715.8422 | 29915.4547 | 25844.5775  | 67089.5804  | 59188.4223 |
| 2.  | MSE ( $\bar{y}_4$ )     | 105446.317 | 117322.785 | 86960.1988 | 91591.5829 | 144668.6619 | 99446.11086 | 62115.4812 |
| 3.  | MSE ( $\bar{y}_5$ )     | 31630.5049 | 32116.9394 | 36588.7598 | 33135.9329 | 33323.84253 | 70175.05245 | 61616.8991 |
| 4.  | MSE ( $\bar{y}_6$ )     | 28836.7787 | 27993.4493 | 34131.6335 | 30228.739  | 26280.09414 | 72475.27807 | 61926.4792 |
|     | Rank MSE( $\bar{y}_7$ ) | 1          | 2          | 1          | 1          | 1           | 1           | 1          |
|     | Rank MSE( $\bar{y}_4$ ) | 4          | 4          | 4          | 4          | 4           | 4           | 4          |
|     | Rank MSE( $\bar{y}_5$ ) | 3          | 3          | 3          | 3          | 3           | 2           | 2          |
|     | Rank MSE( $\bar{y}_6$ ) | 2          | 1          | 2          | 2          | 2           | 3           | 3          |

**Table 6:** Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for LMIEV

| S/N | Populations             | 15          | 16          | 17          | 18          | 19          | 20          | Overall Ranking |
|-----|-------------------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------------|
| 1.  | MSE ( $\bar{y}_7$ )     | 69350.7307  | 70078.14    | 76624.8971  | 724155.3257 | 23664387.66 | 1906665.695 |                 |
| 2.  | MSE ( $\bar{y}_4$ )     | 111018.1903 | 120989.8886 | 500293.3261 | 223833578.3 | 3130175596  | 367888798.1 |                 |
| 3.  | MSE ( $\bar{y}_5$ )     | 72291.75637 | 71855.23721 | 86482.73769 | 1150713.922 | 32801061.58 | 22311430.98 |                 |
| 4.  | MSE ( $\bar{y}_6$ )     | 74927.23977 | 76224.78422 | 95954.58546 | 515671.6613 | 920584.0752 | 2969628.188 |                 |
|     | Rank MSE( $\bar{y}_7$ ) | 1           | 1           | 1           | 2           | 2           | 1           | 1               |
|     | Rank MSE( $\bar{y}_4$ ) | 4           | 4           | 4           | 4           | 4           | 4           | 4               |
|     | Rank MSE( $\bar{y}_5$ ) | 2           | 2           | 2           | 3           | 3           | 3           | 3               |
|     | Rank MSE( $\bar{y}_6$ ) | 3           | 3           | 3           | 1           | 1           | 2           | 2               |

## 5. Percentage Relative Efficiency Analysis For ( Hmaev) Case

6.1 Relative efficiency comparison of the proposed estimator with the reviewed estimators for High and Maximum Extreme Value Cases is given in table 7 to 9.

Table 7: The Relative Efficiency (RE) of the proposed estimator to the reviewed estimators for the twenty simulated populations (measured in percentage) (HMaEV) case.

| S/N | Populations                 | 1        | 2        | 3        | 4        | 5        | 6        | 7        |
|-----|-----------------------------|----------|----------|----------|----------|----------|----------|----------|
| 2.  | $RE(\bar{y}_7 / \bar{y}_4)$ | 2800.974 | 2590.206 | 2874.397 | 2767.547 | 2688.695 | 2979.287 | 3111.432 |
| 3.  | $RE(\bar{y}_7 / \bar{y}_5)$ | 154.0881 | 203.8201 | 174.1303 | 175.8285 | 169.6184 | 198.4446 | 224.2533 |
| 4.  | $RE(\bar{y}_7 / \bar{y}_6)$ | 105.903  | 118.3486 | 111.6314 | 108.6402 | 106.6974 | 122.8343 | 126.0826 |
| 8.  | $RE(\bar{y}_5 / \bar{y}_4)$ | 1817.774 | 1270.829 | 1650.716 | 1574.003 | 1585.144 | 1501.32  | 1387.463 |
| 9.  | $RE(\bar{y}_6 / \bar{y}_4)$ | 2644.848 | 2188.624 | 2574.9   | 2547.444 | 2519.925 | 2425.453 | 2467.773 |
| 10. | $RE(\bar{y}_6 / \bar{y}_5)$ | 145.4993 | 172.2201 | 155.9869 | 161.8449 | 158.9714 | 161.5547 | 177.8623 |

**Table 8:** The Relative Efficiency (RE) of estimators developed by Khan and Shabbir (2013) ratio, AL- Hossain and Khan (2014) regression and AL-Hossain and Khan (2014) ratio with the proposed estimator for the twenty simulated populations (percentage)

| S/N | Populations                 | 8        | 9        | 10       | 11       | 12       | 13       | 14       |
|-----|-----------------------------|----------|----------|----------|----------|----------|----------|----------|
| 2.  | $RE(\bar{y}_7 / \bar{y}_4)$ | 3185.725 | 3433.304 | 3622.812 | 3906.497 | 4166.378 | 3813.837 | 3926.922 |
| 3.  | $RE(\bar{y}_7 / \bar{y}_5)$ | 206.1158 | 157.0593 | 198.2759 | 208.1778 | 227.6207 | 302.4375 | 300.0433 |

|            |                                               |          |          |          |          |          |          |          |
|------------|-----------------------------------------------|----------|----------|----------|----------|----------|----------|----------|
| <b>4.</b>  | <b><math>RE(\bar{y}_7 / \bar{y}_6)</math></b> | 121.041  | 105.9675 | 116.5183 | 126.2429 | 125.8178 | 154.2325 | 155.8988 |
| <b>8.</b>  | <b><math>RE(\bar{y}_5 / \bar{y}_4)</math></b> | 1545.6   | 2185.992 | 1827.157 | 1876.52  | 1830.404 | 1261.033 | 1308.785 |
| <b>9.</b>  | <b><math>RE(\bar{y}_6 / \bar{y}_4)</math></b> | 2631.938 | 3239.958 | 3109.221 | 3094.428 | 3311.437 | 2472.785 | 2518.892 |
| <b>10.</b> | <b><math>RE(\bar{y}_6 / \bar{y}_5)</math></b> | 170.2858 | 148.2145 | 170.1672 | 164.9025 | 180.9129 | 196.092  | 192.4603 |

**Table 9:** The Relative Efficiency (RE) of estimators developed by Khan and Shabbir (2013) ratio, AL- Hossain and Khan (2014) regression and AL-Hossain and Khan

|                                                                                 |                                                   |           |           |           |           |           |           |                 |                                                                          |
|---------------------------------------------------------------------------------|---------------------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------------|--------------------------------------------------------------------------|
| (2014) ratio<br>proposed<br>the twenty-<br>simulated<br>(percentage)<br>Maximum | <b>S/N Populations</b>                            | <b>15</b> | <b>16</b> | <b>17</b> | <b>18</b> | <b>19</b> | <b>20</b> | <b>Average</b>  | with the<br>estimator for<br>one<br>populations<br>for High and<br>Cases |
|                                                                                 | <b>2. <math>RE(\bar{y}_7 / \bar{y}_4)</math></b>  | 5980.514  | 4959.148  | 5977.023  | 6575.026  | 10201.48  | 13952.5   | <b>4675.686</b> |                                                                          |
|                                                                                 | <b>3. <math>RE(\bar{y}_7 / \bar{y}_5)</math></b>  | 166.7048  | 352.8791  | 543.3927  | 407.5045  | 462.7348  | 491.4937  | <b>266.2312</b> |                                                                          |
|                                                                                 | <b>4. <math>RE(\bar{y}_7 / \bar{y}_6)</math></b>  | 106.9121  | 172.3863  | 234.1838  | 173.2953  | 203.2208  | 198.6486  | <b>139.7252</b> |                                                                          |
|                                                                                 | <b>8. <math>RE(\bar{y}_5 / \bar{y}_4)</math></b>  | 3587.488  | 1405.339  | 1099.945  | 1613.485  | 2204.607  | 2838.796  | <b>1768.62</b>  |                                                                          |
|                                                                                 | <b>9. <math>RE(\bar{y}_6 / \bar{y}_4)</math></b>  | 5593.858  | 2876.764  | 2552.278  | 3794.116  | 5019.901  | 7023.709  | <b>3230.413</b> |                                                                          |
|                                                                                 | <b>10. <math>RE(\bar{y}_6 / \bar{y}_5)</math></b> | 155.9269  | 204.7025  | 232.0368  | 235.1503  | 227.7005  | 247.4186  | <b>182.9955</b> |                                                                          |

## 7. Percentage Relative Efficiency Analysis For Low And Minimum Extreme Values ( Lmiev) Cases

7. 1. Relative Efficiency Comparison Of The Proposed Estimator With The Reviewed Estimators Of Khan And Shabbir (2013) Ratio, AL- Hossain And Khan (2014) Regression And AL-Hossain And Khan (2014) Ratio For Lmiev Cases Is Shown In Table 10 To 12.

**Table 10:** The Relative Efficiency (RE) for the twenty simulated populations (measured in percentage) for ( LMiEV) Cases

| S/N | Populations                 | 1        | 2        | 3        | 4        | 5        | 6        | 7        |
|-----|-----------------------------|----------|----------|----------|----------|----------|----------|----------|
| 2.  | $RE(\bar{y}_7 / \bar{y}_4)$ | 1347.723 | 834.8982 | 711.8462 | 939.5984 | 787.8035 | 706.0441 | 1166.393 |
| 3.  | $RE(\bar{y}_7 / \bar{y}_5)$ | 162.3249 | 119.8845 | 113.9064 | 132.8765 | 128.5941 | 117.6756 | 155.8352 |
| 4.  | $RE(\bar{y}_7 / \bar{y}_6)$ | 109.8319 | 99.98373 | 99.9387  | 101.9225 | 101.2601 | 100.0215 | 110.7448 |
| 8.  | $RE(\bar{y}_5 / \bar{y}_4)$ | 830.2629 | 696.4185 | 624.9397 | 707.1217 | 612.6281 | 599.9921 | 748.4786 |
| 9.  | $RE(\bar{y}_6 / \bar{y}_4)$ | 1227.078 | 835.034  | 712.2828 | 921.8757 | 777.9996 | 705.8922 | 1053.226 |
| 10. | $RE(\bar{y}_6 / \bar{y}_5)$ | 147.7939 | 119.9041 | 113.9762 | 130.3701 | 126.9938 | 117.6502 | 140.7156 |

**Table11:** The Relative Efficiency (RE) for the twenty simulated populations (measured in percentage) for ( LMiEV) Cases

| S/N | Populations                 | 14       | 15       | 16     | 17       | 18       | 19       | 20       | Average  |
|-----|-----------------------------|----------|----------|--------|----------|----------|----------|----------|----------|
| 2.  | $RE(\bar{y}_7 / \bar{y}_4)$ | 104.9453 | 160.0822 | 172.65 | 652.9122 | 30909.61 | 13227.37 | 19294.88 | 3653.709 |

  

| S/N | Populations                 | 8        | 9        | 10       | 11       | 12       | 13       | 14       |
|-----|-----------------------------|----------|----------|----------|----------|----------|----------|----------|
| 2.  | $RE(\bar{y}_7 / \bar{y}_4)$ | 366.2647 | 419.0812 | 257.9209 | 306.1681 | 559.7641 | 148.2288 | 104.9453 |
| 3.  | $RE(\bar{y}_7 / \bar{y}_5)$ | 109.8676 | 114.7229 | 108.521  | 110.7653 | 128.9394 | 104.599  | 104.103  |
| 4.  | $RE(\bar{y}_7 / \bar{y}_6)$ | 100.1637 | 99.9936  | 101.2332 | 101.0472 | 101.6851 | 108.0276 | 104.626  |
| 8.  | $RE(\bar{y}_5 / \bar{y}_4)$ | 333.3691 | 365.2988 | 237.6692 | 276.4117 | 434.1296 | 141.7115 | 100.8092 |
| 9.  | $RE(\bar{y}_6 / \bar{y}_4)$ | 365.6661 | 419.108  | 254.7789 | 302.995  | 550.4876 | 137.2138 | 100.3052 |
| 10. | $RE(\bar{y}_6 / \bar{y}_5)$ | 109.6881 | 114.7302 | 107.199  | 109.6173 | 126.8026 | 96.82619 | 99.50008 |

**Table12:** The Relative Efficiency (RE) for the twenty simulated populations (measured in percentage) for LMiEV Cases

|     |                             |          |          |          |          |          |          |          |                 |
|-----|-----------------------------|----------|----------|----------|----------|----------|----------|----------|-----------------|
| 3.  | $RE(\bar{y}_7 / \bar{y}_5)$ | 104.103  | 104.2408 | 102.5359 | 112.8651 | 158.9043 | 138.6094 | 1170.181 | <b>174.9976</b> |
| 4.  | $RE(\bar{y}_7 / \bar{y}_6)$ | 104.626  | 108.041  | 108.7711 | 125.2264 | 71.21009 | 3.890166 | 155.7498 | <b>100.6684</b> |
| 8.  | $RE(\bar{y}_5 / \bar{y}_4)$ | 100.8092 | 153.5696 | 168.3801 | 578.4892 | 19451.71 | 9542.909 | 1648.88  | <b>1912.659</b> |
| 9.  | $RE(\bar{y}_6 / \bar{y}_4)$ | 100.3052 | 148.168  | 158.7278 | 521.3855 | 43406.22 | 340020.6 | 12388.38 | <b>20250.37</b> |
| 10. | $RE(\bar{y}_6 / \bar{y}_5)$ | 99.50008 | 96.48261 | 94.26755 | 90.12882 | 223.1486 | 3563.071 | 751.3207 | <b>324.0093</b> |

## 8. Discussion And Conclusions

This study has proposed improved mixed estimator in the presence of extreme values using two auxiliary variables in single-phase sampling. The theoretical analysis shows that comparing the proposed estimator FEVSA ( $\bar{y}_7$ ) with the ratio estimator of Khan and Shabbir (2013), ( $\bar{y}_4$ ) and the ratio estimator of Al-Hossain and Khan (2014), ( $\bar{y}_5$ ), using their mean square errors, equations 28 and 29 both makes it obvious that FEVSA ( $\bar{y}_7$ ) is superior to these estimators. Lastly, comparing FEVSA ( $\bar{y}_7$ ) with the regression estimator of Al-Hossain and Khan (2014), ( $\bar{y}_6$ ) using equation 30, could not be concluded; but later resolved using empirical analysis.

In the empirical analysis, the R statistical programming language was used to performed exploratory Data Analysis (EDA) for each of the twenty- populations and to summarize the main characteristics (not with visual statistical tools) of the distributions. The function code asymptotically computed the bias, Mean Square Error (MSE) and variance for the proposed and reviewed estimators in each of the twenty stimulated population.

Table 1 through 3 show that the Mean Square Error (MSE) obtained for the proposed estimator is smaller compared to that of the revealed estimators for the cases of HMaEV; this implies that the proposed estimator is more efficient than the reviewed. Likewise, table 4 through 6, agreed with table 1 to 3, that the proposed estimator is superior to the reviewed estimators. Using the ranks for the HMaEV cases, table 3 (the overall rank table for the HMaEV) shows that the proposed estimator FEVSA ( $\bar{y}_7$ ), is ranked first and hence more effective than the reviewed estimators. This is also supported by table 6 (the overall rank table for the LMiEV cases). The empirical analysis revealed that the proposed FEVSA ( $\bar{y}_7$ ), outperformed all the considered estimators for HMaEV and LMiEV cases.

Finally, the function code computed the Relative Efficiency (RE) of the proposed estimator to the reviewed estimators. This answer the question that says by what percentage is the proposed estimator efficient over the reviewed estimators. The percentage relative efficiency Table 9, reveals that the proposed estimator FEVSA,  $\bar{y}_7$  is 6.3803%, 4375.686%, 66.2312% and 39.7252% relatively efficient over  $\bar{y}_4$ ,  $\bar{y}_5$  and  $\bar{y}_6$  respectively for the HMaEV cases. Likewise, table 12 reveals that  $\bar{y}_7$  is 12.361%, 3553.709%, 74.9976% and 0.6684% relatively efficient over  $\bar{y}_4$ ,  $\bar{y}_5$  and  $\bar{y}_6$  respectively for the LMiEV cases. This implies that the proposed estimator  $\bar{y}_7$  is asymptotically more efficient over all the estimators considered in this study irrespective of the type of extreme value case. Therefore, the proposed estimator is recommended subject to the validation of the condition of usage.

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### مقدر مختلط محسن باستخدام متغيرين مساعدين للقيم القصوى والدنيا القصوى الكاملة في أخذ العينات أحادية الطور

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**الخلاصة:** تم إنشاء استخدام متغيرات مساعدة متعددة لتحسين الدقة في مقدرات النسبة والانحدار والمنتج على التوالي. ومع ذلك ، فإن وجود قيم متطرفة في التوزيع يمكن أن يلغي هذه الكفاءة أولاتايو وآخرون. (2020). يمكن أن تكون القيم القصوى قيم صغيرة أو دنيا أو كبيرة أو قصوى. طورت هذه الدراسة مقدر نسبة الانحدار مع متغيرين مساعدين ، معامل الارتباط ومعامل التباين تحت نوعين من القيم القصوى في التوزيع. تتناول هذه الدراسة حالات القيمة القصوى الكاملة التي افترضت أن كل من الدراسة ومتغيرين مساعدين لهما قيم متطرفة موجودة في توزيعاتهما. تم إجراء تحليلات الكفاءة النظرية والتجريبية والنسبة المئوية النسبية للقيم القصوى العالية والقصوى الكاملة (فهميف) وحالات القيم القصوى المنخفضة والدنيا الكاملة (فلميف). أظهر التحليل أن المقدر المطور فعال على المقدرات التي تمت مراجعتها. **الكلمات المفتاحية:** المتغيرات المساعدة ، القيم القصوى ، متوسط الخطأ المربع ، مقدرات النسبة ، مقدر الانحدار ، أخذ العينات أحادي الطور .