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Generalized αii-Closed Sets in Ordinary Topological Spaces

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Abstract

The primary objective of this research is to introduce a novel type of generalized sets, which we have termed generalized αii-closed and generalized αii-open sets. The properties of these sets and their relationships with other types of sets have been rigorously investigated through a series of mathematical proofs, including, if $\Upsilon \subseteq Đ \subseteq (X,\eta)$ is such that Υ is a generalized αii −closed set in (X,η), then Υ is generalized αii −closed set in relation to $(Đ,\eta_{Đ})$), if Υ_1,Υ_2 are generalized αii −closed sets, then so is their intersection, each generalized αii −closed(open) set is a generalized ii −closed(open) set, each generalized α −closed(open) set is a generalized αii − closed(open) set, each semi-generalized closed(open) set is a generalized αii − closed(open) set , a set Υ in (X,η) , is generalized αii −open set when and if only if $U \subseteq IIInt(\Upsilon)$ where U is α −closed set within $U \subseteq \Upsilon$, if $IIInt(\Upsilon) \subseteq Ƶ \subseteq \Upsilon$, wherein, Υ is a generalized αii −open set, then so is Ƶ, and finally, If Υ_1,Υ_2 are generalized αii −open sets , then so is their union.

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1. Introduction

Ashaea, G. S. and Yousif, Y.Y.,[4], introduced some types of mappings in bi-topological spaces in 2021. In 2024,Yousif, Y. Y. and Hussein, N. A.,[20], studied the α–connected fibrewise topological spaces. Askandar,.S.W and Mohammed,. A.A. introduced the concepts of "i-open sets" and " ii–continuity" in 2012 and 2016 [2, 3], which will be used.in this study. In 2019, researchers Mohammed A.A. and Abdullah B.S. introduced "inter-open sets" and " ii –open set" in their research [12]. Aveen and Askandar introduced the concept of "soft generalized αii-closed sets" in soft topological spaces, in 2024. N. Levine[8] made a significant contribution to topology in 1963 by introducing the concept of "semi-open sets" which improved many basic theories of general topology. Njastad, [15] proposed the concept of "α- open sets" in topology in his 1965 paper . It is shown in[15] (see also[16]) that the family of open sets is a subset of the family of α-open sets and the family of α-open sets is always a topology on X, where η^{α} denotes the family of all "α-open sets" of (X,η). In our study, we use these two similar properties and characterization of "i-open sets".

The class of "i-open sets" may be entered together with classes such as, "pre-open sets" [13], "α-open sets" [15], " β-open sets" [1], "regular open sets" [18], "regular semi-open sets" [6], " b-open sets" [7], "pre semi-open sets" [13], t-sets [14], " δ-open sets" [19], and "θ- open sets" [19]. (Ts) It is a topological space(X,η) in this article. Additionally, .interior, closure, $Int(\Upsilon)$, $Cl(\Upsilon)$.We indicate the η elements as" open sets". And (Cs) referred to as" closed sets".

This work consists of three sections. In the second section, we review the fundamental theories of sets and topological spaces. Then, in the third section, we introduce a new concept for these sets, namely, "generalized αii-closed sets" and "generalized αii-open sets", we extended the discussion to include and derive several significant results.

2. Preliminaries

In this section we will give many basic definitions of many concepts of open(closed)sets in topological spaces, also, we review the fundamental theories of sets in topological spaces.

Theorem2.1. [17] Determine that Υ is a subset in (X,η) . Then, there's:

a. $Int\ (\Upsilon^C) =.(Cl\ (\Upsilon))^C$.
b. $Int\ (\Upsilon) = (Cl(\Upsilon^C))^C$.
c. $Cl\ (\Upsilon^C) = (Int\ (\Upsilon))^c$.

Definition2.2. Let Υ be a subset in a topological space (X,η), Υ be known as :

1- " i-open set "[2,3], $for\ short(IOs)$, provided that Os , $\xi \neq \emptyset, X$ exists. and $\Upsilon \subseteq Cl\ (\Upsilon \cap \xi)$.
2- " Semi-open set "[8], $for\ short(SOs)$ if :
a. $\Upsilon \subseteq Cl\ (Int\ (\Upsilon))$.
b. In case Os, $\xi \neq \emptyset, X$ occurs in which $\xi \subseteq \Upsilon \subseteq Cl\ (\xi)$.
3- " α-open set "[15], $for\ short(\alpha Os)$ if " $\Upsilon \subseteq Int(Cl\ (Int\ (\Upsilon)))$.
4- " *Int*-open set "[12], $for\ short(INTOs)$ if an Os , $\xi \neq \emptyset, X$ exists, such that, $Int\ (\Upsilon) = \xi$.
5- " ii-open set"[12], $for\ short(IIOs)$ if Υ is (" i-open" &" inter-open set") .

Definition2.3. Let Υ be a subset in a topological space (X,η), then:

An " i- interior"[2,3] [also an "α- interior"[15],"Semi- interior"[8],"inter- interior"[12], and "ii-interior"[12] of a set Υ is the union From all IOs (separately, (αOs), (SOs), $(INYOs)$, & $(IIOs)$) over X included in Υ, denoted by $IInt\ (\Upsilon)$ [separately, $\alpha Int\ (\Upsilon)$, $SInt\ (\Upsilon)$, $INTInt\ (\Upsilon)$, & $IIInt\ (\Upsilon)$].
" i-closed"[2,3] $for\ short(ICs)$ (separately, " α-closed "[15] $for\ short(\alpha Cs)$, " semi-closed"[8] $for\ short(SCs)$, "inter-closed"[12] $for\ short(INTCs)$, and "ii-closed set"[12] $for\ short(IICs)$) is the whole complement of IOs (separately, αOs, SOs, $INTOs$, & $(IIOs)$).
The "i-closure"[2,3] (also the "α- closure"[15], "semi- closure"[8], "inter- closure"[12], and "ii-closure"[12]) of Υ is indicated by $ICl\ (\Upsilon)$ (separately, $\alpha Cl\ (\Upsilon)$, $SCl\ (\Upsilon)$, $INTCl\ (\Upsilon)$, and $IICl\ (\Upsilon)$) is the intersection From all ICs (separately, αCs, SCs, $INTCs$, & $IICs$) over X comprising Υ.

Definition 2.4. " generalized closed set", $\boldsymbol{for\ short(GCs)}$ if $\boldsymbol{Cl(\Upsilon) \subseteq \xi}$ wherein, $\boldsymbol{\Upsilon \subseteq \xi}$ and $\boldsymbol{\xi}$ is a $\boldsymbol{Os}$ in $\boldsymbol{(X,\eta)}$ is taken into consideration by A sets, $\boldsymbol{\Upsilon}$ in$\boldsymbol{(X,\eta)}$.The term " generalized open set" $\boldsymbol{(GOs)}$ refers to $\boldsymbol{GOs'S}$ complement**[9]**.

Definition 2.5. A subset Υ in a topological space (X,η) is named:

1. A" generalized α - closed set" $for\ short\ (G\alpha Cs)$ if $Cl\ (\Upsilon) \subseteq \xi$ is so as to $\Upsilon \subseteq \xi$ and ξ together form an αOs in (X,η) [11].
2. The" $\alpha -$ generalized closed set", $for\ short\ (\alpha GCs)$, if $\alpha Cl(\Upsilon) \subseteq \xi$ is So as to $\Upsilon \subseteq \xi$ and ξ together form an Os in (X,η) [10].

3. The " semi- generalized closed set" $for\ short(SGCs)$ is def. as follows : $SCl\ (\Upsilon) \subseteq \xi$ where $\Upsilon \subseteq \xi$ and ξ are an SOs in (X,η) [5].

Definition2.6. Let Υ be a subset in a topological space (X,η), then:

The complement of $(G\alpha Cs)$ (resp., (αGCs), and $(SGCs)$) is known as " generalized α-open" [11] $for\ short$ $(G\alpha Os)$ (resp., " $\alpha -$ generalized open"[10] $for\ short$ (αGOs), and " semi-generalized open"[5] $for\ short(SGCs)$. Each $G\alpha Cs$ (resp., αGCs, $SGCs$ and $GSCs$ in (X,η) acquired $G\alpha Cs(X)$) (resp., $\alpha GCs(X)$, $SGCs(X)$ and $GSCs(X)$).

Definition2.7. Consider Υ be any subset in (X,η). Let" $\eta_{\Upsilon} = \{(G \cap \Upsilon): G \in \eta\}$" be a topology on Υ. This topology is called the relative topology of η on Υ, and $\{\Upsilon, \eta_{\Upsilon}\}$ is called the subspace of (X,η) [17].

Proposition2.8. Each Os is a IOs [3].

Proposition2.9. Each Cs is a ICs [3].

Proposition2.10. Each SOs is a $IIOs$ [12].

Propositin2.11. Each SCs is a $IICs$[12].

Proposition2.12. Each αOs is a SOs [3].

Proposition2.13. Each αCs is a SCs [3].

Proposition2.14. Each αOs is a $IIOs$ [12].

Propostion2.15. Each αCs is a $IICs$ [12].

3. Generalized αii −Closed Sets

Definition 3.1. Let Υ be a subset in a topological space (X,η), then Υ considers:

i. Generalized αii-closed set, in short $(G\alpha IICs)$ If $IICl\ (\Upsilon) \subseteq \xi$ is such that $\Upsilon \subseteq \xi$ and ξ is a αOs in (X,η). $G\alpha IICs\ (X)$ Includes all $G\alpha IICs$ in *X*.

ii. Generalized ii-closed set, $in\ short(GIICs)$ since $IICl\ (\Upsilon) \subseteq \xi$ " wherein $\Upsilon \subseteq \xi$ and ξ is a Os in (X,η). The of all $GIICs$ in X is designated.by $GIICs\ (X)$.

Remark 3.2. Let Υ be a set in (X,η), then Υ considers as a generalized αii - open set, $(G\alpha IIOs)$ when its complement Υ^c is a $G\alpha IICs$. The collection of every $G\alpha IIOs$ is denoted by $G\alpha IIOs(X)$.

Corollary 3.3. A set Υ in (X,η), is $G\alpha IIOs$ when and if only if $U \subseteq IIInt(\Upsilon)$ where U is a αCs within $U \subseteq \Upsilon$.

Proof: Assume that Υ is $G\alpha IIOs$ with $U \subseteq \Upsilon$ and U is a αCs. Then Υ^c is $G\alpha IICs$ and U^c is αOs with, $\Upsilon^c \subseteq U$, $IICl(\Upsilon)^c \subseteq U^c$. Henceforth,$U \subseteq IIInt(\Upsilon)$.

Assume, in reverse, that $U \subseteq IIInt(\Upsilon)$ with $U \subseteq \Upsilon$ and U is αCs. Think of M as a αOs that contains Υ^c.

Consequently, $M^c \subseteq IIInt(\Upsilon)$, then, $IICl(\Upsilon)^c \subseteq M$. Henceforth, Υ^c is $G\alpha IICs$. Which implies Υ be $G\alpha IIOs$■

Theorem3.4. Each $IICs$ is a $G\alpha IICs$.

Proof: Assume that a $IICs$, Υ in (X,η) and ξ is a αOs wherein, $\Upsilon \subseteq \xi$, we get, $IICl(\Upsilon) = \Upsilon \subseteq \xi$. Henceforth, Υ is a $G\alpha IICs$■

Corollary 3.5. Each $IIOs$ is a $G\alpha IIOs$.

Proof: Assume that a $IIOs$, Υ in (X,η) , We obtain, Υ^c is $GIICs$, by "(Theorem3.4)" we get, Υ^c is $G\alpha IICs$. Henceforth, Υ is a $G\alpha IIOs$■

Theorem3.6. If $\Upsilon \subseteq Ƶ \subseteq IICl\ (\Upsilon)$, wherein, Υ is a $G\alpha IICs$, then so is Ƶ .

Proof: Assume $Ƶ \subseteq \xi$ and ξ be a αOs in (X,η), then $\Upsilon \subseteq \xi$. Since Υ is a $G\alpha IICs$,
$Ƶ \subseteq IICl\ (\Upsilon)$, we can conclude that "$IICl\ (Ƶ) \subseteq IICl(\Upsilon)$". We obtain " $IICl\ (Ƶ) \subseteq IICl\ (\Upsilon) \subseteq \xi$ ". We obtained that " $IICl\ (Ƶ) \subseteq \xi$ ". Henceforth, Ƶ is a $G\alpha IICs$■

Corollary 3.7. If $IIInt(\Upsilon) \subseteq Ƶ \subseteq \Upsilon$, wherein, Υ is a $G\alpha IIOs$, then so is Ƶ.

Proof: Consider $IIInt(\Upsilon) \subseteq Ƶ \subseteq \Upsilon$, $Ƶ \subseteq \xi$ then,

$\Upsilon^c \subseteq Ƶ^c \subseteq IICl(\Upsilon^c)$ with Υ^c is $G\alpha IICs$. Thus, $Ƶ^c$ is $G\alpha IICs$ "(Theorem3.6)". Henceforth, Ƶ is $G\alpha IIOs$■

Theorem3.8. If $\Upsilon \subseteq Đ \subseteq (X,\eta)$ is such that Υ is a $G\alpha IICs$ in (X,η), then Υ is $G\alpha IICs$ in relation to $(Đ,\eta_{Đ})$.

Proof: Consider $\Upsilon \subseteq \xi$ and ξ represent a αOs in Đ..Since $\Upsilon \subseteq Đ$, gives us $\Upsilon \subseteq (\xi \cap Đ)$, ξ is a αOs in $(Đ,\eta_{Đ})$. Consequently., there is an αOs M in X where $\xi = (M \cap Đ)$, then, $\Upsilon \subseteq \xi \subseteq M$ with Υ representing a $G\alpha IICs$ in (X,η) gives us $IICl\,(\Upsilon) \subseteq M$ by $IICl\,(\Upsilon) \cap Đ$ is a " ii-closure" of Υ in $(Đ,\eta_{Đ})$, giving us $IICl\,(\Upsilon) \cap Đ \subseteq \xi$. From this point on, Υ is a $G\alpha IICs$ w. r. t. $(Đ,\eta_{Đ})$■

Theorem3.9. If Υ_1, Υ_2 are $G\alpha IICs$, then so is their intersection.

Proof: Let Υ_1, Υ_2 as a $G\alpha IICs$ in (X,η) and ξ_1, ξ_2 are any αOs where in $\Upsilon_1 \subseteq \xi_1$ and $\Upsilon_2 \subseteq \xi_2$, then, $IICl\,(\Upsilon_1) \subseteq \xi_1$ and $IICl\,(\Upsilon_2) \subseteq \xi_2$, $(IICl\,(\Upsilon_1 \cap \Upsilon_2) \subseteq IICl\,((\Upsilon_1) \subseteq IICl\,(\Upsilon_2))$,
we get : $IICl(\Upsilon_1 \cap \Upsilon_2) \subseteq (\xi_1 \cap \xi_2)$ and given that the intersection of any two αOs is a αOs, put $(\xi_1 \cap \xi_2) = \xi$, with ξ is a αOs, $IICl(\Upsilon_1 \cap \Upsilon_2) \subseteq \xi$. Henceforth, $(\Upsilon_1 \cap \Upsilon_2)$ is a $G\alpha IICs$■

Corollary 3.10. If Υ_1, Υ_2 are $G\alpha IIOs$, then so is their union.

Proof: Take note of Υ_1, Υ_2 as a $G\alpha IIOs$ in (X,η). We obtain, $\Upsilon_1^c, \Upsilon_2^c$ are $G\alpha IIOs$. "(Theorem3.9)" gives us the result that $(\Upsilon_1^c \cap \Upsilon_2^c) = (\Upsilon_1 \cap \Upsilon_2)^c$ is a $G\alpha IICs$. Therefore, $(\Upsilon_1 \cup \Upsilon_2)$is now a $G\alpha IIOs$■

The union of two generalized αii-closed sets is not necessary to be generalized αii-closed .Indeed,

Example3.11. Let $X = \{3,6,9\}$, $\eta = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$ **is a topology on X.**

Where , $\boldsymbol{\Upsilon_1} = \{3\}$ **,** $\Upsilon_2 = \{6\}$, $\Upsilon_3 = \{3,6\}$

Let $\Upsilon_4 = \{6,9\}$, $\Upsilon_5 = \{3,9\}$, $\Upsilon_6 = \{9\}$

$Os(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$.

$Cs(X) = \{\emptyset, X, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.

$\alpha Os(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$.

$IOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.

$INTOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.

$IIOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.

$IICs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.

$G\alpha IICs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.

Thus, Υ_1 and Υ_2 are $G\alpha IICs$,obviously, $(\Upsilon_1 \cup \Upsilon_2) = \Upsilon_3$ is a αOs and $(\Upsilon_1 \cup \Upsilon_2) \subseteq \Upsilon_3$ But, $II - Cl(\Upsilon_3) = X \nsubseteq \Upsilon_3$. Hence $(\Upsilon_1 \cup \Upsilon_2) = \Upsilon_3$ is not $G\alpha IICs$.

Also the intersection of two generalized αii- open sets is not necessary to be generalized αii- open .Indeed, $G\alpha IIOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.

Thus, Υ_4 and Υ_5 are s , but $(\Upsilon_4 \cap \Upsilon_5) = \Upsilon_6$ is not $G\alpha IIOs$.

Theorem3.12. Each $G\alpha IICs$ is a $GIICs$.

Proof: Consider Υ_1 is a $G\alpha IICs$ in (X,η) and ξ is any Os wherein, $\Upsilon \subseteq \xi$, then $IICl\,(\Upsilon) \subseteq \xi$ since each Os is αOs with ξ is a Os . Henceforth, Υ is a $GIICs$■

Corollary 3.13. Each $G\alpha IIOs$ is a $GIIOs$.

Proof: If we take Υ to be a $G\alpha IIOs$ in (X,η), we obtain Υ^c to be an $G\alpha IICs$, which suggests Υ^c to be $GIICs$ "(Theorem3.12)." From now on, Υ^c is a $GIIOs$■

"The converse is not true**:** "

Example3.14. Let $X = \{3,6,9\}$, $\eta = \{\emptyset, X, \Upsilon_1, \Upsilon_2\}$ is a topology on X.

Where , $\boldsymbol{\Upsilon_1} = \{3\}$, $\boldsymbol{\Upsilon_2 = \{3,9\}}$

Let $\Upsilon_3 = \{6,9\}$, $\Upsilon_4 = \{6\}$, $\Upsilon_5 = \{3,6\}$, $\Upsilon_6 = \{9\}$

$Os(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2\}$.

$Cs(X) = \{\emptyset, X, \Upsilon_3, \Upsilon_4\}$.
$\alpha Os(X.) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_5\}$.
$IOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_5\}$.
$INTOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_5\}$.
$IIOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_5\}$.
$IICs (X) = \{\emptyset, X, \Upsilon_3, \Upsilon_4, \Upsilon_6\}$.
$G\alpha IICs(X) = \{\emptyset, X, \Upsilon_3, \Upsilon_4, \Upsilon_6\}$.
$GIICs(X) = \{\emptyset, X, \Upsilon_3, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.
Thus, Υ_5 is not $G\alpha IICs$ because,it is a αOs ,with $\Upsilon_5 \subseteq \Upsilon_5$ .But, $II - Cl(\Upsilon_5) = X \nsubseteq \Upsilon_5$,
Υ_5 is a $GIICs$ **.**
By the definition of $G\alpha IIOs$ and the definition of $GIIOs$, we have
$G\alpha IIOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_5\}$.
$GIIOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_5, \Upsilon_6\}$.
Obviously, Υ_6 is not $G\alpha IIOs$.But Υ_6 is a .

Theorem3.15. Each $G\alpha Cs$ is a $G\alpha IICs$.
Proof: Assume that Υ is a $G\alpha Cs$ in (X,η) and ξ is any αOs wherein, $\Upsilon \subseteq \xi$, $IICl(\Upsilon) \subseteq Cl(\Upsilon) \subseteq \xi$ will that mean $IICl(\Upsilon) \subseteq \xi$ with O is a αOs . Henceforth, Υ is a $G\alpha IICs$■

Corollary 3.16. Each $G\alpha Os$ is a $G\alpha IIOs$.
Proof: Suppose that Υ is a $G\alpha Os$ in (X,η). Then, Υ^c is an $G\alpha Cs$, and Υ^c is $G\alpha IICs$ "(Theorem3.15)". Henceforth, Υ is a $G\alpha IIOs$■
"The converse is not true**:** "

Example 3.17. Let $X = \{3,6,9\}$ **,** $\eta = \{\emptyset, X, \boldsymbol{\Upsilon_1}, \boldsymbol{\Upsilon_2}, \boldsymbol{\Upsilon_3}\}$ is a topology on X.
Where , $\Upsilon_1 = \{3\}$ **,** $\Upsilon_2 = \{6\}$, $\Upsilon_3 = \{3,6\}$
Let $\Upsilon_4 = \{6,9\}$, $\Upsilon_5 = \{3,9\}$, $\Upsilon_6 = \{9\}$
$Os(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$.
$Cs(X) = \{\emptyset, X, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.
$\alpha Os(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$.
$IOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.
$INTOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.
$IIOs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4, \Upsilon_5\}$.
$IICs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.
$G\alpha IICs(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.
$G\alpha Cs(X) = \{\emptyset, X, \Upsilon_4, \Upsilon_5, \Upsilon_6\}$.
Thus, Υ_1 is $G\alpha IICs$ because,it is a αOs ,with $\Upsilon_1 \subseteq \Upsilon_1$ .And $II - Cl(\Upsilon_1) = \Upsilon_1 \subseteq \Upsilon_1$,
Υ_1 is not a $G\alpha Cs$ because, $Cl(\Upsilon_1) = \Upsilon_5 \nsubseteq \Upsilon_1$ **.**
By the definition of $G\alpha IIOs$ and the definition of $G\alpha Os$, we have
$G\alpha IIOs(X) = \{\emptyset, X, \Upsilon_4, \Upsilon_5, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$.
$G\alpha Os(X) = \{\emptyset, X, \Upsilon_1, \Upsilon_2, \Upsilon_3\}$.
Obviously, Υ_4 is a $G\alpha IIOs$.But Υ_4 is not a $G\alpha Os$.

Theorem3.18. Each $SGCs$ is a $G\alpha CIIs$.
Proof: Consider Υ is an $SGCs$ in (X,η) and $\Upsilon \subseteq \xi$ where ξ is any αOs and SOs, since, each SCs is a $IICs$ "(Proposition2.11)", then $IICl(\Upsilon) \subseteq SCl(\Upsilon) \subseteq \xi$. Henceforth, Υ is a $G\alpha IICs$■

Corollary 3.19. Each $SGOs$ is a $G\alpha IIOs$.
Proof: If we assume that Υ is a $SGOs$ in (X,η), Then, Υ^c is an $SGCs$, we conclude that Υ^c is $G\alpha IICs$. "(Theorem3.18)". Henceforth, Υ is a $G\alpha IIOs$■

4. Conclusions

According to the above we concluded that The relationship among αCs, $IICs$, SCs, and Cs (respectively, the relationship among, αOs, $IIOs$, SOs, and Os)determines the relationship among $G\alpha IICs$, $G\alpha Cs$, GCs, $GIICs$ and $SGCs$(respectively, the relationship among $G\alpha IIOs$, $G\alpha Os$, GOs, $GIIOs$ and $SGOs$).

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المجاميع المغلقة من النمط – αii المعممة في الفضاءات التبولوجية الاعتيادية

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المستخلص:

يهدف هذا البحث إلى تقديم نوع جديد من المجاميع المعممة، أطلقنا عليه اسم المجاميع المغلقة والمفتوحة من النمط – αii المعممة. وتم التحقق من خصائص هذه المجاميع وعلاقتها بأنواع أخرى من المجاميع من خلال مجموعة من البراهين الرياضية، ,ومنها أنه اذا كانت اية مجموعة Υ في الفضاء الكلي (X,η) مغلقة من النمط – αii معممة فأنها تكون ايضا مغلقة من النمط – αii معممة في الفضاء الجزئي $(Đ,\eta_{Đ})$، تقاطع اية مجموعتين مغلقتين من النمط – αii معممتين Υ_1,Υ_2 يكون ايضا مجموعة مغلقة من النمط – αii معممة، كل مجموعة مغلقة(مفتوحة) من النمط – αii معممة تكون مغلقة(مفتوحة) من النمط –ii معممة، كل مجموعة مغلقة(مفتوحة) من النمط– α معممة تكون مغلقة(مفتوحة) من النمط–αii معممة، كل مجموعة شبه مغلقة(مفتوحة) معممة تكون مغلقة(مفتوحة) من النمط–αii معممة، المجموعة Υ في الفضاء التبولوجي (X,η) تكون مفتوحة من النمط–αii معممة اذا وفقط اذا كان $U \subseteq II -Int(\Upsilon)$ حيث $U \subseteq \Upsilon$ و U مجموعة مغلقة من النمط–α، اذا كان $II - Int(\Upsilon) \subseteq Ƶ \subseteq \Upsilon$ بحيث ان Υ مجموعة مفتوحة من النمط–αii معممة، عندئذ Ƶ ايضا مجموعة مفتوحة من النمط–αii معممة، واخيرا، اتحاد مجموعتان مفتوحتان من النمط–αii معممة في الفضاء التبولوجي ، يكون ايضا مجموعة مفتوحة من النمط–αii معممة.