

Finding the general Solution for some extraordinary Differential Equation

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الملخص

يتضمن هذا البحث دراسة هدفين ، الهدف الاول كيفية تحويل معادلة تفاضلية

كسرية والتى لها الصيغة $AD^q y + BD^\alpha y + Cy(x) = 0$ ، $m-1 < \alpha < m$ باستخدام مؤثرات (Riemann-Liouville) الى معادلة تفاضلية اعتيادية . الهدف الثانى تطبيق لطريقة ادومين التركيبية لحل المعادلة التفاضلية الكسرية كطريقة تتبعية لطريقة تحويل لاپلاس.

Abstract

This paper has a dual purpose. One purpose is to introduce a method consists of repeatedly applying Riemann-Liouville operator of appropriate order until the equation

$AD^q y + BD^\alpha y + Cy(x) = 0$, $(m-1 < \alpha < m)$, $(q \text{ is integer})$ is transformed into an ordinary differential equation.A second purpose is to apply the Adomian decomposition method for the solution of fractional differential equation an alternative method of Laplace transform.

1. Introduction

The idea of differentiation to an arbitrary order started in 1695 when L'Hopital asked Liebniz what would happen with $d^n y/dx^n$ when $n=1/2$. Subsequently, the topic started with the misnomer fractional calculus because n can be rational,irrational or complex. After a half century of controversy due to the lack of precise definition of fractional derivative. In 1819 Lacrois the integral nth derivative of $\frac{D_x^n}{x^m}$ by induction and found that $\frac{D_x^{1/2}}{x} = 2\sqrt{\frac{x}{\pi}}$.

Bertram [6] introduced a method of solving $f(x) + \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt = g(x)$ $0 < \alpha < 1$

Where $\alpha = 2/3$ and $g(x) = x^2$.Which is conceptually different than that volterra, and Laplace transforms .Al-Shamani [4] studied a general

method of solving integral equations of the form

$$f(x) + \int_0^x I^\alpha f(x) = g(x) \quad 0 < \alpha < 1 \quad (1.1)$$

by repeatedly applying Riemann-Liouville operators of an appropriate order until (1.1) is transformed into an ordinary differential equation in $f(x)$, where $g(x)$ is a known function and where $f(0)$ and the derivatives $f^{(J)}(0)$, ($J=1,2,3\dots$) are assumed to be finite.

Elzaidi [7] solved the extraordinary differential equation

$$y^\alpha(x) = \frac{y}{x}, \quad (x \neq 0, \alpha \in R^+)$$

with initial condition $y_0(x) = \frac{cx^{\alpha-1}}{\Gamma(\alpha)}$. By using Laplace transform and

Adomian decomposition

method. Murad [10] give a general method of transformed fractional differential equation of the form

$$D^\alpha[y(x) - y_0(x)] = \beta y(x) + f(x) \quad 0 < \alpha < 1, \beta < 0 \quad \text{in to ordinary}$$

differential equation by repeatedly applying suitable operators. Saha and Bera [9] solved a extraordinary differential equation of the form

$$\frac{dy}{dx} + \frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}} - 2y = 0 \quad \text{,with initial condition } y(0)=C_1$$

using the Adomian decomposition method. Then the solution obtained by this method is verified with that of the transformed ordinary differential equation derived from the original fractional differential equation. Similar relationship involving at least one integro-differential on non-integer order may be termed as on extraordinary differential equation. Such an equation is solved when an explicit expression for f is exhibited. As with ordinary differential equations often involve integrals and contain arbitrary constants as discussed in [8] these types of equations are also known as fractional differential equations. The application of extraordinary differential equation is now available in many physical and technical areas.

2. Definitions and Lemmas

Definition 2.1[8]

The Gamma function is denoted by Γ and is defined by the integral

$$\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} dx \quad \text{for } \alpha > 0$$

Definition 2.2[5]

Let f a function which is defined a.e on $[a,b]$. for $\alpha > 0$, we define

$I_a^\alpha f = \frac{1}{\Gamma(\alpha)} \int_a^b (b-t)^{\alpha-1} f(t) dt$ provided that the integral (Lebesgue) exists.

Definition 2.3[5]

If $\alpha < 0$, and n is the smallest positive integer such that $\alpha + n > 0$ we define

$$I_a^\alpha f = D_x^n I_a^{\alpha+n} f, \text{ at } x=b \text{ provided that } I_a^{\alpha+n} f \text{ and its first } (n-1)$$

derivatives exist in a segment. $|b-x| < h$, where h is a appositive real, and the n th derivative exists at $x=b$.

Corollary 2.4[3]

If $\alpha \in R$ and $f(x)$ is defined a.e on $a \leq x \leq b$ we define $f^{(\alpha)}(x) = I_a^{x-\alpha} f$

for all $x \in [a,b]$, provided that $I_a^{x-\alpha} f$ exists.

Lemma 2.5[3]

If $\alpha > 0$, n is the smallest integer $> \alpha$; $f(x)$ is in $L(a,b)$ and we define

$I_a^{x-1-\alpha} f$ exists and continuous on $[a,b]$ then $I_a^{a-i-k} f = K_i$ exists for

$i=1,2,\dots,n$; $I_a^{x-\alpha} f$ exists a.e on $a \leq x \leq b$ is in $L(a,b)$ and

$I_a^\alpha I_a^{-\alpha} f = f(x) - \sum_{p=1}^n \frac{K_p (x-a)^{\alpha-p}}{\Gamma(\alpha-p+1)}$, a.e on $a \leq x \leq b$. Furthermore, the

equality holds everywhere on $a < x \leq b$, if in addition $f(x)$ is continuous on $a < x \leq b$.

Lemma 2.6[5]

Let $\alpha, \beta \in R, \beta > -1$. If $x > a$ then

$$I_a^\alpha (t-a)^\beta = \begin{cases} \frac{(x-a)^{\alpha+\beta} \Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)} & \alpha+\beta \neq \text{negative integer} \\ 0 & \alpha+\beta = \text{negative integer} \end{cases}$$

Lemma 2.7[3]

The relation $\frac{x}{a} \alpha \frac{x}{a} \beta f = \frac{x}{a} \alpha + \beta f$ holds if

(i) $\operatorname{Re}(\alpha) > 0, \operatorname{Re}(\beta) > 0$, and $f(x)$ is in $C^{(0)}$ on $[a, b]$.

(ii) $\operatorname{Re}(\alpha) > 0, \operatorname{Re}(\beta) \leq 0$ or $\operatorname{Re}(\beta+m) > 0$, such that $\beta \neq -m$,

and $f(x)$ is in $C^{(m)}$ on $[a, b]$

(iii) $\operatorname{Re}(\alpha) \leq 0, \operatorname{Re}(\alpha+n) > 0, \operatorname{Re}(\beta) > 0$, and $f(x)$ is in $C^{(n)}$ on $[a, b]$.

(iv) $\operatorname{Re}(\alpha) \leq 0, \operatorname{Re}(\beta) \leq 0, \beta \neq -m$, and $f(x)$ is in $C^{(m+n)}$ on $[a, b]$.

when $\beta = -m$ in (ii) and (iv), then

$$\frac{x}{a} \alpha \frac{x}{a} -m f = \frac{x}{a} \alpha -m f - \sum_{p=0}^{m-1} \frac{(x-a)^{\alpha-m+p}}{\Gamma(\alpha+p-m+1)} f^{(p)}(a)$$

3. Helping Transformation

3.1 Theorem

The extraordinary differential equation

$$A \frac{d^q y}{dx^q} + B \frac{d^\alpha y}{dx^\alpha} + Cy(x) = 0 \quad m-1 < \alpha < m, (m=1, 2, \dots) \quad (3.1)$$

Can be transformed to an ordinary differential equation of the form

$$B^n \frac{x}{0}^{-n\alpha} y(t) + (-1)^{n+1} C^n y(x) = \sum_{k=1}^n B^{n-k+1} C^{k-1} (-1)^{k-1} \sum_{p=1}^n \frac{K_p x^{(k-n)\alpha-p}}{\Gamma((k-n)\alpha-p+1)} + \\ A \sum_{k=1}^n B^{n-k} C^{k-1} (-1)^k \frac{x}{0}^{(k-n)\alpha-q} y(t).$$

Provided that $n\alpha$ is any positive integer greater or equal to one, k_1, k_2, k_3, \dots are constant and q is integer number.

Proof

By using Corollary(2.4) we can write (3.1) as

$$A \frac{x}{0}^{-q} y(t) + B \frac{x}{0}^{-\alpha} y(t) + Cy(t) = 0 \quad (3.2)$$

Operate (3.2) with $\frac{x}{0}^\alpha$ we get

$$A \frac{x}{0} \alpha \frac{x}{0}^{-q} y(t) + B \frac{x}{0} \alpha \frac{x}{0}^{-\alpha} y(t) + C \frac{x}{0} \alpha y(t) = 0$$

By using lemma (2.7)(ii)and Lemma(2.5) we get

$$\begin{aligned}
 & A \int_0^x \alpha^{-q} y(t) + B y(x) - B \sum_{p=1}^n \frac{K_p x^{\alpha-p}}{\Gamma(\alpha-p+1)} + C \int_0^x \alpha^{-p} y(t) = 0 \\
 & B y(x) + C \int_0^x \alpha^{-p} y(t) = B \sum_{p=1}^n \frac{K_p x^{\alpha-p}}{\Gamma(\alpha-p+1)} - A \int_0^x \alpha^{-q} y(t) \quad (3.3)
 \end{aligned}$$

n is smallest integer number , provided $\alpha+n > 0$

Operate (3.3) with $\int_0^x {}^{-1}$ getting

$$B \int_0^x {}^{-1} y(t) + C \int_0^x {}^{-1} \int_0^x \alpha^{-p} y(t) = \int_0^x {}^{-1} B \sum_{p=1}^n \frac{K_p t^{\alpha-p}}{\Gamma(\alpha-p+1)} - A \int_0^x {}^{-1} \int_0^x \alpha^{-q} y(t)$$

By using lemma(2.7)(iii) and lemma(2.6) getting

$$B \int_0^x {}^{-1} y(t) + C \int_0^x {}^{-1} \alpha^{-1} y(t) = B \sum_{p=1}^n \frac{K_p x^{\alpha-p-1}}{\Gamma(\alpha-p)} - A \int_0^x {}^{-1} \alpha^{-q-1} y(t)$$

$$\int_0^x {}^{-1} \alpha^{-1} y(t) = \frac{1}{C} \left[B \sum_{p=1}^n \frac{K_p x^{\alpha-p-1}}{\Gamma(\alpha-p)} - A \int_0^x {}^{-1} \alpha^{-q-1} y(t) - B \int_0^x {}^{-1} y(t) \right] \quad (3.4)$$

Operate (3.3) with $\int_0^x {}^{-1}$ getting

$$B \int_0^x {}^{-1} y(t) + C \int_0^x {}^{-1} 2\alpha^{-1} y(t) = B \sum_{p=1}^n \frac{K_p x^{2\alpha-p-1}}{\Gamma(2\alpha-p)} - A \int_0^x {}^{-1} 2\alpha^{-q-1} y(t) \quad (3.5)$$

In equation (3.5) substitute for $\int_0^x {}^{-1} y$ which is obtained from equation

(3.4) getting

$$B \left[B \sum_{p=1}^n \frac{K_p x^{\alpha-p-1}}{\Gamma(\alpha-p)} - A \int_0^x {}^{-1} \alpha^{-q-1} y(t) - B \int_0^x {}^{-1} y(t) \right] + C \int_0^x {}^{-1} 2\alpha^{-1} y(t) =$$

$$= B \sum_{p=1}^n \frac{K_p x^{2\alpha-p-1}}{\Gamma(2\alpha-p)} - A \int_0^x {}^{-1} 2\alpha^{-q-1} y(t)$$

$$B^2 \int_0^x y(t) - C^2 \int_0^x y(t) = B^2 \sum_{p=1}^n \frac{K_p x^{\alpha-p-1}}{\Gamma(\alpha-p)} - CB \sum_{p=1}^n \frac{K_p x^{2\alpha-p-1}}{\Gamma(2\alpha-p)}$$

$$-AB \int_0^x y(t) + CA \int_0^x y(t) \quad (3.6)$$

Operate equation (3.3) with \int_0^x we getting

$$\int_0^x y(t) = \frac{1}{B} \left[-C \int_0^x y(t) + B \sum_{p=1}^n \frac{K_p x^{3\alpha-p-1}}{\Gamma(3\alpha-p)} - A \int_0^x y(t) \right] \quad (3.7)$$

In equation (3.7) substitution for $\int_0^x y(t)$ which is obtained from (3.6)

$$B^3 \int_0^x y(t) + C^3 \int_0^x y(t) = B^3 \sum_{p=1}^n \frac{K_p x^{\alpha-p-1}}{\Gamma(\alpha-p)} - CB^2 \sum_{p=1}^n \frac{K_p x^{2\alpha-p-1}}{\Gamma(2\alpha-p)} \\ + BC^2 \sum_{p=1}^n \frac{K_p x^{3\alpha-p-1}}{\Gamma(3\alpha-p)} - AB^2 \int_0^x y(t) + ABC \int_0^x y(t) - AC^2 \int_0^x y(t)$$

Continuing in this process we get

$$B^n \int_0^x y(t) + (-1)^{n+1} C^n \int_0^x y(t) = \sum_{k=1}^n B^{n-k+1} C^{k-1} (-1)^{k-1} \sum_{p=1}^n \frac{K_p x^{k\alpha-p-1}}{\Gamma(k\alpha-p)} + \\ A \sum_{k=1}^n B^{n-k} C^{k-1} (-1)^k \int_0^x y(t) \quad (3.8)$$

Equation (3.8) is an ordinary differential equation with respect to y , finally

operate an each term of equation (3.8) with $\int_0^x 1-n\alpha$, we get

$$B^n \int_0^x y(t) + (-1)^{n+1} C^n y(x) = \sum_{k=1}^n B^{n-k+1} C^{k-1} (-1)^{k-1} \sum_{p=1}^n \frac{K_p x^{(k-n)\alpha-p}}{\Gamma((k-n)\alpha-p+1)} +$$

$$A \sum_{k=1}^n B^{n-k} C^{k-1} (-1)^k \int_0^x (k-n)\alpha - q y(t) dt. \quad (3.9)$$

Which is required equation and this complete proof.

Remark 3.1

Using Corollary (2.4), equation (3.9) can be written as

$$B^n y^{(n\alpha)}(x) + (-1)^{n+1} C^n y(x) = \sum_{k=1}^n B^{n-k+1} C^{k-1} (-1)^{k-1} \sum_{p=1}^n \frac{K_p x^{(k-n)\alpha-p}}{\Gamma((k-n)\alpha-p+1)} + A \sum_{k=1}^n B^{n-k} C^{k-1} (-1)^k \int_0^x (k-n)\alpha - q y(t) dt. \quad (3.10)$$

Example:

To illustrate the procedure, we will specify $\alpha = \frac{1}{2}$, $q = 1$ and $A=B=1$,

$C=-2$, in equation (3.1)

$$\text{Getting } \frac{dy}{dx} + \frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}} - 2y = 0 \quad (3.11)$$

Saha and Bara [9] transformed the equation(3.11) to an ordinary differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = \frac{C}{2}x^{-\frac{3}{2}} \quad \text{and for calculation see[9].}$$

Corollary 3.1

The solution of (3.10) is sum of the complementary solution y_c and particular solution y_p .The particular solution for each term of the right side of (3.9) can be obtained by using the method of variation of parameters.

4. Solution of an extraordinary differential equation

In this section we get some results about the solution of an extraordinary differential equation.

4.1 Theorem

The extraordinary differential equation

$$AD^q y + BD^\alpha y + Cy(x) = 0 \quad m-1 < \alpha < m \quad (4.1)$$

with initial condition $y(0)=C_1, y'(0)=C_2, \dots, y^{(q-1)}(0)=C_{q-1}$ and q is integer., has a solution given by

$$y(x) = C_1 - \frac{C_1}{A} \left[C \frac{x^q}{\Gamma(q+1)} + B \frac{x^{q-\alpha}}{\Gamma(q-\alpha+1)} \right] + \frac{C_1}{A^2} \left[C^2 \frac{x^{2q}}{\Gamma(2q+1)} + 2CB \frac{x^{2q-\alpha}}{\Gamma(2q-\alpha+1)} \right. \\ \left. + B^2 \frac{x^{2q-2\alpha}}{\Gamma(2q-2\alpha+1)} \right] - \frac{C_1}{A^3} \left[C^3 \frac{x^{3q}}{\Gamma(3q+1)} + 3C^2 B \frac{x^{3q-\alpha}}{\Gamma(3q-\alpha+1)} \right]$$

$$\begin{aligned}
 & + 3B^2C \frac{x^{3q-2\alpha}}{\Gamma(3q-2\alpha+1)} + B^3 \frac{x^{3q-3\alpha}}{\Gamma(3q-3\alpha+1)} \Big] + \frac{C_1}{A^4} \left[C^4 \frac{x^{4q}}{\Gamma(4q+1)} + 4C^3B \frac{x^{4q-\alpha}}{\Gamma(4q-\alpha+1)} \right. \\
 & + 6B^2C^2 \frac{x^{4q-2\alpha}}{\Gamma(4q-2\alpha+1)} + 4CB^3 \frac{x^{4q-3\alpha}}{\Gamma(4q-3\alpha+1)} + B^4 \frac{x^{4q-4\alpha}}{\Gamma(4q-4\alpha+1)} \Big] \\
 & - \frac{C_1}{A^5} \left[C^5 \frac{x^{5q}}{\Gamma(5q+1)} + 5C^4B \frac{x^{5q-\alpha}}{\Gamma(5q-\alpha+1)} + 10B^2C^3 \frac{x^{5q-2\alpha}}{\Gamma(5q-2\alpha+1)} + 10B^3C^2 \frac{x^{5q-3\alpha}}{\Gamma(5q-3\alpha+1)} + \right. \\
 & \quad \left. 5CB^4 \frac{x^{5q-4\alpha}}{\Gamma(5q-4\alpha+1)} + B^5 \frac{x^{5q-5\alpha}}{\Gamma(5q-5\alpha+1)} \right] + \dots
 \end{aligned}$$

Proof.(Adomian Decomposition method)

we suppose that $L = \frac{d^q}{dx^q}$. Therefore, by the Adomian decomposition method, we can write

$$\begin{aligned}
 D^q y(x) &= -\frac{1}{A} \left[BD^\alpha y + Cy(x) \right] \\
 \frac{d^q y}{dx^q} &= -\frac{1}{A} \left[B \frac{d^\alpha y}{dx^\alpha} + Cy(x) \right]
 \end{aligned} \tag{4.2}$$

we operate equation (5.2) by L^{-1} getting

$$y(x) = -\frac{1}{A} \left[BL^{-1} \left(\frac{d^\alpha y}{dx^\alpha} \right) + CL^{-1}(y) \right]$$

$$\text{This implies that } y(x) = -\frac{1}{A} \left[C \frac{d^{-q} y}{dx^{-q}} + B \frac{d^{\alpha-q} y}{dx^{\alpha-q}} \right]$$

In the light of the Adomian decomposition method, we assume $y(x) = y_0(x) + y_1(x) + y_2(x) \dots$ To be the solution of equation (4.1), where

$$y_1(x) = -\frac{1}{A} \left[C \frac{d^{-q} y_0}{dx^{-q}} + B \frac{d^{\alpha-q} y_0}{dx^{\alpha-q}} \right], \text{ where } y_0 = C_1$$

by using Corollary (2.4) we get

$$y_1(x) = -\frac{1}{A} \left[C \frac{x}{I_0^q} y_0 + B \frac{x}{I_0^{q-\alpha}} y_0 \right] = -\frac{1}{A} \left[C \frac{x}{I_0^q} C_1 + B \frac{x}{I_0^{q-\alpha}} C_1 \right]$$

by using lemma(2.6)

$$y_1(x) = -\frac{C_1}{A} \left[C \frac{x^q}{\Gamma(q+1)} + B \frac{x^{q-\alpha}}{\Gamma(q-\alpha+1)} \right]$$

$$y_2(x) = -\frac{1}{A} \left[C \frac{d^{-q} y_1}{dx^{-q}} + B \frac{d^{\alpha-q} y_1}{dx^{\alpha-q}} \right]$$

by using Corollary (2.4) we get

$$y_2(x) = -\frac{1}{A} \left[C {}_{I^q}^x y_1 + B {}_{I^q}^{x-\alpha} y_1 \right]$$

$$y_2(x) = \frac{C_1}{A^2} \left[C^2 {}_{I^q}^x \frac{x^q}{\Gamma(q+1)} + CB {}_{I^q}^x \frac{x^{q-\alpha}}{\Gamma(q-\alpha+1)} + BC {}_{I^q}^{x-\alpha} \frac{x^q}{\Gamma(q+1)} \right. \\ \left. + B^2 {}_{I^q}^{x-\alpha} \frac{x^{q-\alpha}}{\Gamma(q-\alpha+1)} \right]$$

by using lemma(2.6)

$$y_2(x) = \frac{C_1}{A^2} \left[C^2 \frac{x^{2q}}{\Gamma(2q+1)} + 2CB \frac{x^{2q-\alpha}}{\Gamma(2q-\alpha+1)} + B^2 \frac{x^{2q-2\alpha}}{\Gamma(2q-2\alpha+1)} \right]$$

$$y_3(x) = -\frac{1}{A} \left[C \frac{d^{-q} y_2}{dx^{-q}} + B \frac{d^{\alpha-q} y_2}{dx^{\alpha-q}} \right]$$

by using the Corollary (2.4) we get

$$y_3(x) = -\frac{1}{A} \left[C {}_{I^q}^x y_2 + B {}_{I^q}^{x-\alpha} y_2 \right]$$

$$y_3(x) = -\frac{C_1}{A^3} \left[C^3 \frac{x^{3q}}{\Gamma(3q+1)} + 3C^2B \frac{x^{3q-\alpha}}{\Gamma(3q-\alpha+1)} + 3B^2C \frac{x^{3q-2\alpha}}{\Gamma(3q-2\alpha+1)} + B^3 \frac{x^{3q-3\alpha}}{\Gamma(3q-3\alpha+1)} \right]$$

$$y_4(x) = \frac{C_1}{A^4} \left[C^4 \frac{x^{4q}}{\Gamma(4q+1)} + 4C^3B \frac{x^{4q-\alpha}}{\Gamma(4q-\alpha+1)} + 6B^2C^2 \frac{x^{4q-2\alpha}}{\Gamma(4q-2\alpha+1)} + 4CB^3 \frac{x^{4q-3\alpha}}{\Gamma(4q-3\alpha+1)} + B^4 \frac{x^{4q-4\alpha}}{\Gamma(4q-4\alpha+1)} \right]$$

$$y_5(x) = -\frac{C_1}{A^5} \left[C^5 \frac{x^{5q}}{\Gamma(5q+1)} + 5C^4B \frac{x^{5q-\alpha}}{\Gamma(5q-\alpha+1)} + 10B^2C^3 \frac{x^{5q-2\alpha}}{\Gamma(5q-2\alpha+1)} + \right.$$

$$\left. 10B^3C^2 \frac{x^{5q-3\alpha}}{\Gamma(5q-3\alpha+1)} + 5CB^4 \frac{x^{5q-4\alpha}}{\Gamma(5q-4\alpha+1)} + B^5 \frac{x^{5q-5\alpha}}{\Gamma(5q-5\alpha+1)} \right]$$

and so on. Therefore the solution of (4.1) is

$$y(x) = C_1 - \frac{C_1}{A} \left[C \frac{x^q}{\Gamma(q+1)} + B \frac{x^{q-\alpha}}{\Gamma(q-\alpha+1)} \right] + \frac{C_1}{A^2} \left[C^2 \frac{x^{2q}}{\Gamma(2q+1)} + 2CB \frac{x^{2q-\alpha}}{\Gamma(2q-\alpha+1)} \right]$$

$$\begin{aligned}
 & + B^2 \frac{x^{2q-2\alpha}}{\Gamma(2q-2\alpha+1)} \Bigg] - \frac{C_1}{A^3} \left[C^3 \frac{x^{3q}}{\Gamma(3q+1)} + 3C^2B \frac{x^{3q-\alpha}}{\Gamma(3q-\alpha+1)} \right. \\
 & + 3B^2C \frac{x^{3q-2\alpha}}{\Gamma(3q-2\alpha+1)} + B^3 \frac{x^{3q-3\alpha}}{\Gamma(3q-3\alpha+1)} \Bigg] + \frac{C_1}{A^4} \left[C^4 \frac{x^{4q}}{\Gamma(4q+1)} + 4C^3B \frac{x^{4q-\alpha}}{\Gamma(4q-\alpha+1)} \right. \\
 & + 6B^2C^2 \frac{x^{4q-2\alpha}}{\Gamma(4q-2\alpha+1)} + 4CB^3 \frac{x^{4q-3\alpha}}{\Gamma(4q-3\alpha+1)} + B^4 \frac{x^{4q-4\alpha}}{\Gamma(4q-4\alpha+1)} \Bigg] \\
 & - \frac{C_1}{A^5} \left[C^5 \frac{x^{5q}}{\Gamma(5q+1)} + 5C^4B \frac{x^{5q-\alpha}}{\Gamma(5q-\alpha+1)} + 10B^2C^3 \frac{x^{5q-2\alpha}}{\Gamma(5q-2\alpha+1)} + 10B^3C^2 \frac{x^{5q-3\alpha}}{\Gamma(5q-3\alpha+1)} + \right. \\
 & \quad \left. 5CB^4 \frac{x^{5q-4\alpha}}{\Gamma(5q-4\alpha+1)} + B^5 \frac{x^{5q-5\alpha}}{\Gamma(5q-5\alpha+1)} \right] + \dots \quad (4.3)
 \end{aligned}$$

Example:

To illustrate the procedure we will specify the extraordinary differential equation $Dy(x) + D^{\frac{1}{2}}y(x) - 2y(x) = 0$ where the initial condition is $y(0) = C_1$

by using equation(4.3) we get

$$y(x) = C_1 \left[1 + 3x + \frac{11x^2}{2} - 2 \frac{\sqrt{x}}{\sqrt{\pi}} - \frac{20x^{\frac{3}{2}}}{3\sqrt{\pi}} - \frac{56x^{\frac{5}{2}}}{5\sqrt{\pi}} + \dots \right] \text{ Which is}$$

the same solution in [9].

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