Determination of Backbending in $^{122-130}$Ba Even-Even Isotopes

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الخلاصة

أستخدم نموذج البوزونات المتفاعلة $IBM-1$ التحديد $O(6)$ ، γ-اللامستقر في تحديد الانحناء الخلفي في نظائر الباريوم ضعيفة التشوه $^{122-130}Ba$ الزوجية-الزوجية. وقد اظهر تطبيق هذا التحديد نجاحاً في تحديد الانحناء الخلفي في مستويات الطاقة للنظائر المدروسة من خلال التطابق الجيد مع النتائج العملية.

Abstract

The γ-unstable O(6) limit of the interacting boson model IBM-1 has been applied successfully to determine the backbending in weakly deformed $^{122-130}$ Ba even-even isotope. The application of this limit has showed successes in determining the backbending in the energy levels of the isotopes under consideration through the good coincidence with the experimental results.

Introduction:

The backbending phenomenon occurs due to the rapid increase of the moment of inertia with rotational frequency toward the rigid value [1]. When the rotational energy exceeds the energy needed to break a pair of nucleon, the unpaired nucleon goes into a different orbit causing a change in the moment of inertia [2], other proposed explanation such as rotational alignment [3], and centri-fugal stretching, [4] along with the former, could be described in terms of band crossing [5].

The main purpose of the present work is to investigate the backbending effect in $^{122-130}$ Ba even-even isotopes, using γ-unstable O(6) limit of the interacting boson model IBM-1 [6].

Several studies have been performed to investigate the backbending effect in some even-even Ba isotopes [7 and 8] .

Theory:

Yrast levels and backbending

The lower energy level for each spin is called yrast level [9]. One of the interesting observations made on yrast bands is the presence of small and sudden changes in the moment of inertia on a plot of E_J as a function of J(J+1). The sudden change are usually too insignificant to be noticeable. However if the moment of inertia is plotted against the square of the frequency of the rotation, a local variation in the moment of inertia around a significant high spin would be occured.

The rotational energies are given by [2]:

$$E_J = \frac{\hbar^2}{2\Im} J(J+1) \qquad (1)$$

$\Im$ Is the moment of inertia and J is the spin of the state .
where
The energy of a transition from state J to the next lower state J-2 is given by [9]:

$$E_J - E_{J-2} = \frac{\hbar^2}{2\Im}(4J-2) \qquad (2)$$

and the local value of the moment of inertia will be:

$$\frac{2\Im}{\hbar^2} = \frac{4J-2}{E_J - E_{J-2}} \qquad (3)$$

The rotational frequency ω is not a quantity that can be measured but may be inferred by analogy with classical rotational frequency through the relation [9]:

$$\hbar\omega = \frac{dE}{d\sqrt{J(J+1)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

For a K=0 band, the usual case for the yrast band in even-even nuclei; the value may be approximated by the difference between E_J and E_{J-2} to:

$$\hbar\omega \cong \left|\frac{\Delta E}{\Delta\sqrt{J(J+1)}}\right|_{J-2}^{J} \xrightarrow[J\to\infty]{} \frac{1}{2}(E_J - E_{J-2}) \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

γ-unstable O (6) limit

A unified description of collective nuclear states was proposed by Arima & Iachello [10] where each nucleon pair is considered as a boson [11], the low-lying collective states can be classified according to the totally symmetric irreducible representation [N] of the group SU (6).

There are a dynamical symmetries results from the degeneracy of the unitary group SU(6) to three limits with three analytical solution [11]. γ-unstable O(6) limit is the one among these limits ,where the group SU(6) degenerate to the chain [12] :

$$SU(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2) \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

The wave function which describes this limit is given by the quantum numbers $\left|N\sigma\tau\nu_\Delta LM_L\right\rangle$ where σ is used to characterize the totally symmetric irreducible representations of O (6). This quantum number may take the following values [13]

$$\sigma = N, N-2, \ldots\ldots\ldots, 0 or 1 \; for N = even..or...odd \ldots\ldots\ldots\ldots(7)$$

and τ characterizes the totally symmetric irreducible representation of O (5) where [12]

$$\tau = \sigma, \sigma - 1, - - -- , 0 \ldots\ldots\ldots\ldots \ldots\ldots\ldots\ldots(8)$$

The quantum number υ_Δ counts d-boson triplets that coupled to zero angular momentum; and finally L and M_L are the total angular momentum and its Z-component.

The values of L which are contained in each representation τ of O(5) are obtained by partitioning τ as [12]

$$\tau = 3\upsilon_\Delta + \lambda \qquad \upsilon_\Delta = 0,1 \ldots\ldots\ldots\ldots (9)$$

and taking

$$L = 2\lambda, 2\lambda - 2, \ldots\ldots, \lambda + 1, \lambda \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Note that the absence of L= 2λ-1 value and the Hamiltonian of this group is given by [13]

$$H_{O(6)} = aP^{\wedge +}.P^{\wedge} + bT_3^{\wedge}.T_3^{\wedge} + cL^{\wedge}.L^{\wedge} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

This Hamiltonian has an eigen values [14]

$$E(\sigma,\upsilon,L) = K_3[N(N+4) - \sigma(\sigma+4) + K_4\upsilon(\upsilon+3) + K_5 L(L+1) \ldots\ldots\ldots\ldots (12)$$

where K_3 , K_4 and K_5 are the strength parameters for each term.

Results and Discussions:

γ- unstable O(6) limit of the interacting boson model IBM-1 has been used to determine the energy levels of the yrast band for A= 122-130 even-even Ba isotopes. The first term of eqn. (13) has no significant effect since σ=N for the yrast band .The double solution of eqn. (13) for each isotope gives two values for each parameters K_4 and K_5 for both bands, the ground state band and the other band, which gathered to form the yrast levels.

Table (1) shows the values of K_4 and K_5 for the ground state band and the other band for each isotope under consideration. So through knowing the values of K_4 and K_5, a determinations of the yrast levels can be performed by using egn.(12) for each isotopes. Table (2) shows the available measured and present calculation of the yrast levels where a good agreement has been found.

Fig.(1) shows the experimental and the caculated energy levels E_J in (keV) against J (J+1) for each isotopes and it is obvious that the backbending phenomenon is observed for each isotopes under consideration except ^{122}Ba isotope.

A calculation of $\frac{2\Im}{\hbar^2}$ has been done by using eqn.(3), and $(\hbar\omega)^2$ is calculated using eqn.(5). Fig.(2) shows the experimental and the calculated $\frac{2\Im}{\hbar^2}$ in keV against $(\hbar\omega)^2$ in $(\text{keV})^2$ for all used isotopes and the backbending phenomenon is clearly observed for all isotopes except ^{122}Ba.

conclusion:

The present work suggests that the interacting boson model IBM-1, for γ-unstable O(6) limit is a successful tool to study the yrast levels in the transition nuclei as in $^{122-130}$Ba isotopes. Using this model gives a fairly accurate description of the backbending phenomenon in $^{122-130}$Ba even-even isotopes except ^{122}Ba, and this and this ascribes to the ultimate available values of the energy levels of ^{122}Ba isotope, while the energy levels for the other isotopes ($^{124-130}$Ba) are more available to determine the backbending phenomenon. The appearance
of the backbending depending on the amount of deformation of the isotope.

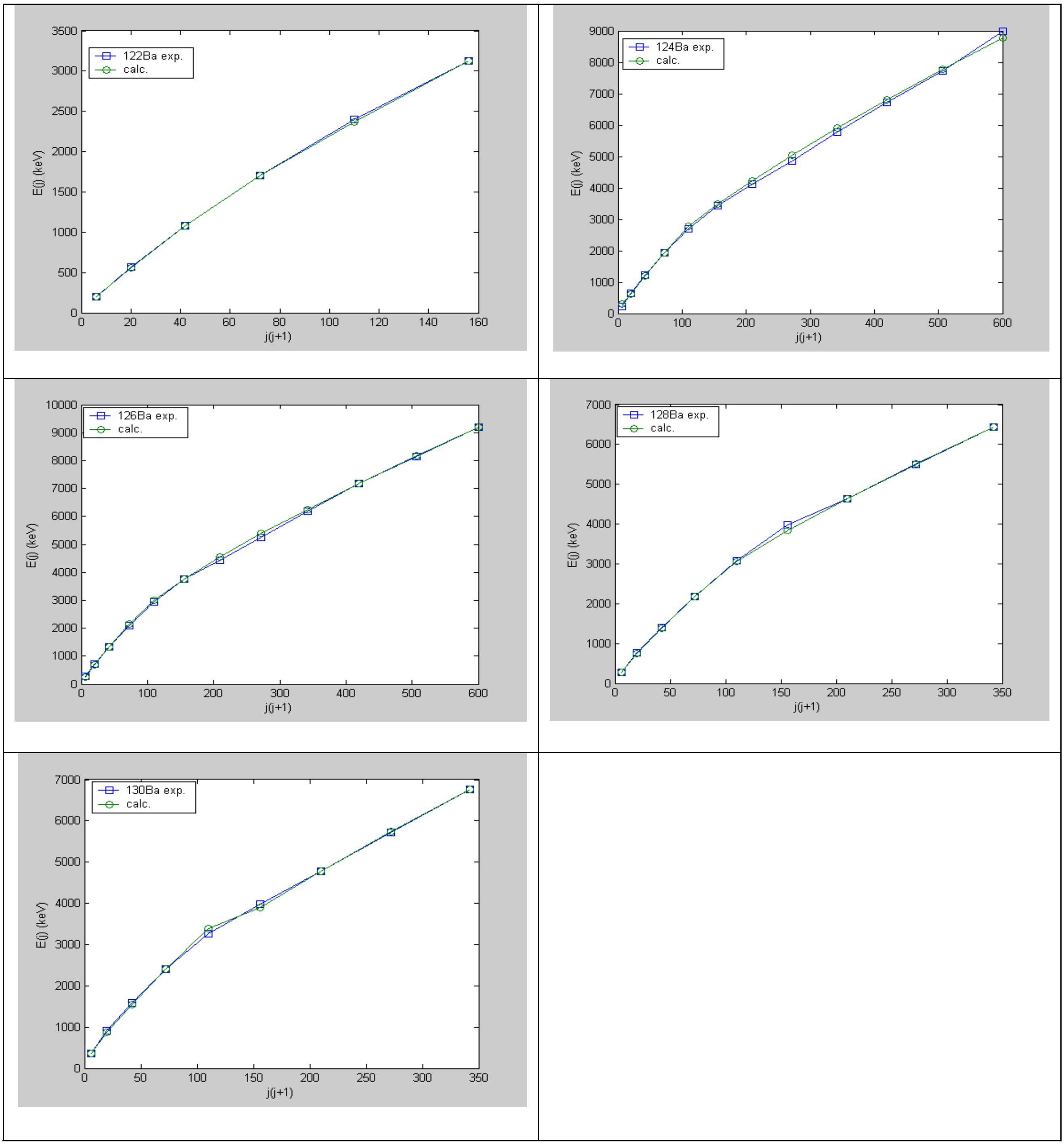
Isotope	g.s band		other.bands	
	K_4	K_5	K_4	K_5
^{122}Ba	28.9	13.4	85.2	-9.466
^{124}Ba	41.85	10.433	165.1	-35.12
^{126}Ba	45.9	12.067	188.83	-41.34
^{128}Ba	61.0	6.667	188.39	-40.673
^{130}Ba	94.45	-3.466	172.1	-34.6

Table (1) the value of K_4 and K_5 parameters in keV for $^{122-130}$Ba even-even isotopes

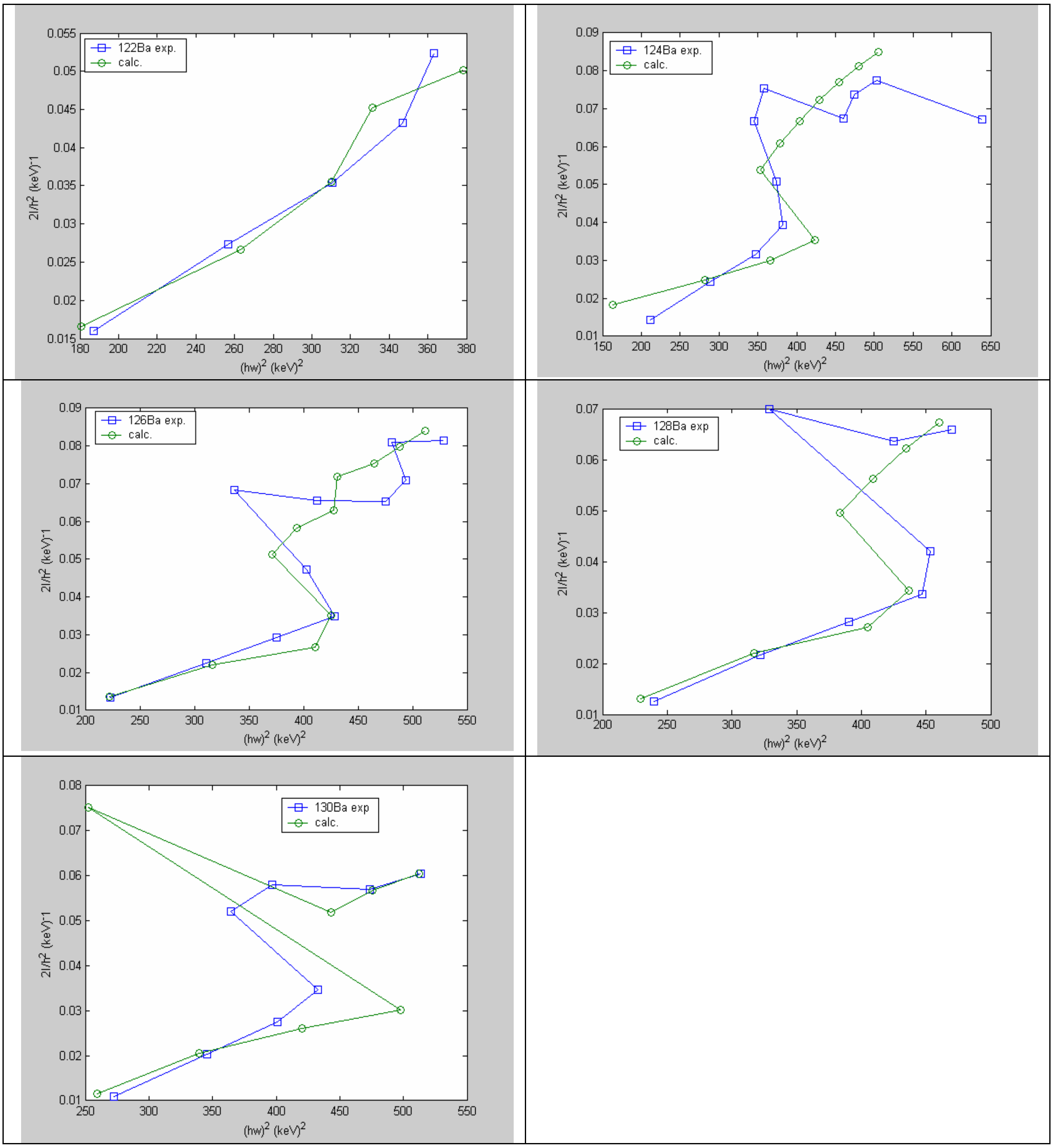
	^{122}Ba		^{124}Ba		^{126}Ba		^{128}Ba		^{130}Ba	
	Eexp (a)	Ecalc	Eexp(b)	Ecalc	Eexp (c)	Ecalc	Eexp (d)	Ecalc	Eexp (e)	Ecalc
2+	196	196	229.84	229.84	265	256	284	284	357	357
4+	570	557	651.65	627.0	711	700	763	743	902	875
6+	1083	1083	1228.38	1191.0	1333	1333	1407	1378	1593	1554
8+	1704	1703	1923.23	1923.23	2084	2154	2188	2188	2395	2395
10+	2398	2366	2687.4	2771	2942	3005	3082	3061	3260	3390
12+	3124	3123	3436.2	3477	3747	3747	3988	3828	3989	3895
14+			4125.9	4234	4420	4535	4646	4646	4783	4781
16+			4842.5	5042	5245	5391	5496	5515	5730	5733
18+			5763.2	5900	6195	6253	6436	6436	6757	6757
20+			6711.1	6810	7183	7183				
22+			7716.4	7770	8145	8159				
24+			8994.4	8781	9202	9182				

Table (2): Comparison between the measured and calculated energy yrast levels in (keV)

a) Ref (15)
b) Ref (16)
c) Ref (17)
d) Ref (18)
e) Ref (19)



Fig(1): The energy levels E(j) in (keV) against j(j+1)



Fig(2): The $2\xi/\hbar^2$ in $(keV)^{-1}$ agaist $(\hbar\omega)^2$ in $(keV)^2$

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