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Soft Semi Totally Continuity in Soft Topological Spaces

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Abstract

The soft semi-totally continuous mappings that we have introduced in this study are stronger than the soft totally continuous mappings and provide fresh thoughts for soft continuous mappings between several soft topological spaces. By using evidence and guidelines to clarify and explain it, the relationships between these notions and several other concepts of soft mappings have been studied. Also, several of these functions' characteristics have been looked into. Moreover, soft semi-totally open mappings have been shown and investigated. Additionally, we defined soft continuous mappings that depend on soft i-open sets and looked at how they are related to soft continuous mappings that depend on soft semi-open sets. In this paper we proved that each soft semi-totally continuous mapping is soft totally continuous , each soft strongly semi continuous mapping is soft strongly i-continuous, each soft totally semi continuous mapping is soft totally i-continuous, the composition of two soft semi totally open mappings is soft semi totally open mapping, and the composition of two soft semi totally continuous mappings is soft semi totally continuous mapping.

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Introduction

In 1963, the ideas of semi-open sets were introduced ([13]). Askandar, S.W. 2012 ([4]) introduced the idea of i-open sets in conventional topological spaces., in 2020 and 2022([5,6]), Askandar and Mohammed introduced the concepts of soft i-open sets and soft i-continuous mappings. Soft sets and their characteristics have been discussed by Molodtsov, and a variety of experts in 1999, 2003, 2009, 2011, 2014, and 2015 ([15], [14], [2], [20], [19], [10]). Chen, B., and Kannan, K., respectively, provided concepts for soft semi-open sets and soft α -open sets in soft topological spaces in 2013 ([8]) and 2012 ([11]). In 2014, Ozturk, T. Y. and Bayramov, S., ([18]) incorporated soft point notions into the study ([7]). Soft point principles from the paper ([21]) have been employed in this work.

As a generalization of totally continuous functions, T. M. Nour ([16]) presented the idea of totally semi-continuous functions, and numerous features of totally soft semi-continuous functions were established. This paper introduces and studies soft semi-totally continuity, a new generalization of soft strong continuity that is more robust than soft totally continuity. Additionally, these functions' fundamental characteristics and soft semi-totally continuous functions' preservation theorems are examined. Also introduced and researched are soft semi-totally open functions in soft topological spaces. We concluded by introducing the sot i-totally continuous mappings and contrasting them with the ideas previously presented.

Material and methods

Definition 1. Let X represent the initial universe, *P(X)* represent its power set, *T* represent its parameter set, and ($\emptyset \neq A \subseteq T$). A pair *(K,A)* is referred to as a soft set (*sS*) over *X*, where *K* is thought of as mapping $K: A \rightarrow P(X)$. A parameterized collection with *X* subdivisions is known as a soft set. If " $t \in A$, that is, " $K_A = \{K(t): t \in A \subseteq T, K: A \rightarrow P(X)\}$" then *K(t)* is termed a soft set *(K,A)* group of t-approximate factors in Specific $t \notin A$. The family of all these soft sets over *X* is designated as $SS(X_A)$ ([15]).

Definition 2. If $\emptyset_T$, X_T, the union of any number of soft sets in τ, as well as the intersection of any two soft sets in τ, all belong to τ, then τ is said to be soft topology on *X*. The triple consider (X, τ, T) and is a soft topological space (*sTs*). Soft open sets are referred to as the embodiment of τ . If the complement $(K,T)^c$ of *(K,T)* in $(X, \tau, T)$is known as a soft closed set. The collection of all soft closed sets over *X* is denoted by the symbol *(sCs)* ([20]).

Definition 3. The soft closure of *(K,T)* is defined as the intersection of all the soft closed sets that include *(K,T)*, and is denoted by *Cl(K,T)* containing *(K, T)* ([20]). The *(K,T)* soft interior, denoted by *Int(K,T)*([9]), is the union of the complete soft open sets contained in *(K,T).*

Definition 4. Let (X,τ,T) and (Y,ρ,T') consistently imply soft topological spaces written as *(sTs)* and $f_{pu} : SS(X_T) \rightarrow SS(Y_{T'})$, while mappings include $u : X \rightarrow Y$, and $p : T \rightarrow T'$.

1. For a soft set *(F,J)* in (X,τ,T), $(f_{pu}(W,J),Z)$, $Z=p(J) \subseteq T'$ is a soft set in (Y,ρ,T') given by:

$$f_{pu}(W,J)(\beta) = \begin{cases} u\left(\bigcup_{\alpha \in p^{-1}(\beta) \cap A} W(\alpha)\right), if\ p^{-1}(\beta) \cap J \neq \emptyset \\ \phi, otherwise \end{cases}$$

For $\beta \in Z \subseteq$T', $(f_{pu}(W,J), Z)$ is referred to as a soft image of a soft set *(W,J)*.([1]).

2. *If (M,I)* is a soft set in (Y,ρ,T'), where, $I \subseteq T'$,

Then $(f^{-1}_{pu}(M,I),L)$, $L=p^{-1}(I)$, is a soft set in (X,τ,T), defined by:

$$f^{-1}_{pu}(M,I)(\alpha) = \begin{cases} u^{-1}\left(M\left(P(\alpha)\right)\right), if\ p(\alpha) \in I \\ \phi, \quad otherwise \end{cases}$$

For $\alpha \in M \tilde{\subseteq}$T. $(f^{-1}_{pu}(M,I),L)$ is referred to as a soft set *(M,I)* inverted image. ([1]).

Definition 5. If (W,T) is a *(sS)* in(X,τ,T), it would be considered:

i. Soft semi-open set written as *(sSOs)* if: a. $(W,T) \tilde{\subseteq} Cl(Int(W,T))$. *b.* If a *sOs* $(O,T) \neq \phi, X$ exists and $(O,T) \tilde{\subseteq} (W,T) \tilde{\subseteq} Cl(O,T)$ is present ([8]).

ii. If a *sOs* $(O,T) \neq \phi, X$ exists wherein $(W,T) \tilde{\subseteq} Cl((W,T) \tilde{\cap} (O,T))$, soft i-open set written as *(sIOs)* is used ([5]).

The complement *sSOs* is called a soft semi-closed set written as (*sSCs*). The soft semi-closure of (W,T) and designated $SCl(W,T)$ is the intersection of all *sSCs* over X containing (W,T). A soft semi-interior of a soft set (W,T) is the union of all *sSOs* over X contained in (W,T), and it is represented by $SInt(W,T)$. The collection of all *sOs* , *sSOs* , (*sCs* , *sSCs*) in (X,τ,T) are denoted by $sOs(X_T)$)($sSOs(X_T)$, , $sCs(X_T)$, $sSCs(X_T)$.

The term "soft clopen set" written as (*(sCOs)*) refers to a set that can be soft open and closed. ($sCOs(X_T)$) stands for the family of all soft clopen sets. A soft semi-clopen set is written as *(sSCOs)* is a set that can be soft semi open and semi closed. $(sSCOs(X_T))$ designates the family of all soft semi-clopen sets.

Definition 6. A $f_{pu} : SS(X_T) \rightarrow SS(Y_{T'})$ soft mapping with $p : T \rightarrow T'$ and $u : X \rightarrow Y$ is known as:

(i) Continuous [21] written as $(sCONm)$ if $f_{pu}^{-1}(D,T') \tilde{\in} sOs(X_T)\ \forall (D,T') \tilde{\in} sOs(Y_{T'})$.

(ii) Semi-continuous [12] written as $(sSCONm)$ if $f_{pu}^{-1}(D,T') \tilde{\in} sSOs(X_T)\ \forall (D,T') \tilde{\in} sOs(Y_{T'})$.

(iii) Totally continuous written as $(sTCONm)$ if $f_{pu}^{-1}(D,T') \tilde{\in} sCOs(X_T)\ \forall (D,T') \tilde{\in} sOs(Y_{T'})$.

(iv) Strongly continuous written as $(sSTRCONm)$ if $f_{pu}^{-1}(D,T') \tilde{\in} sCOs(X_T)\ \forall (D,T') \tilde{\subseteq} Y_{T'}$.

(v) Totally semi-continuous written as $(sTSCONm)$ if, $f_{pu}^{-1}(D,T') \tilde{\in} sSCOs(X_T)\ \forall (D,T') \tilde{\in} sOs(Y_{T'})$.

(vi) Strongly semi-continuous written as $(sSTRSCONm)$ if, $f_{pu}^{-1}(D,T') \tilde{\in} sSCOs(X_T)\ \forall (D,T') \tilde{\subseteq} Y_{T'}$.

(vii) Irresolute written as $(sIREm)$ [12] if, $f_{pu}^{-1}(D,T') \tilde{\in} sSOs(X_T)\ \forall (D,T') \tilde{\in} sSOs(Y_{T'})$.

(viii) Semi-open written as $(sSOm)$ [12] if, $f_{pu}(D,T) \tilde{\in} sSOs(Y_{T'})\ \forall (D,T) \tilde{\in} sOs(X_T)$.

(ix) Semi-closed written as $(sSCm)$ [12]if, $f_{pu}(D,T) \tilde{\in} sSCs(Y_{T'})\ \forall (D,T) \tilde{\in} sCs(X_T)$.

Example 1. Let $X=\{11,13,15\}$, $Y=\{2,4,6\}$, $T=\{t,q\}$, $T'=\{t',q'\}$,

$\tau=\{\phi_T, X_T, (\beta_1,T), (\beta_2,T), (\beta_3,T)\}$, $\rho=\{\phi_{T'}, Y_{T'}, (D_1,T')\}$.

Where, $(\beta_1,T)=\{(t,\{11\}),(q,\{11\})\}$, $(\beta_2,T)=\{(t,\{13\}),(q,\{13\})\}$,

$(\beta_3,T)=\{(t,\{11,13\}),(q,\{11,13\})\}$, $(D_1,T')=\{(t',\{2,4\}),(q',\{2,4\})\}$.

$sOs(X_T)=\{\phi_T, X_T, (\beta_1,T), (\beta_2,T), (\beta_3,T)\}$, $sOs(Y_{T'})=\{\phi_{T'}, Y_{T'}, (D_1,T')\}$,

$\{\phi_T, X_T, (\beta_1,T), (\beta_2,T), (\beta_3,T)\}, \{(t,\{11,15\}),(q,\{11,15\})\}, \{(t,\{13,15\}),(q,\{13,15\})\}\} \tilde{\in} sSOs(X_T)$.

$sCOs(X_T)=\{\phi_T, X_T\}$

Describe the mapping $f_{pu}: SS(X_T) \to SS(Y_{T'})$ at this time, where $p: T \to T'$ and $u: X \to Y$ are distinguished by: $p(t)=t'$, $p(q)=q'$, $u(11)=2, u(13)=6, u(15)=4$.

Plainly, f_{pu} is not $sCONm$ since (D_1,T') is an sOs in Y but $f_{pu}^{-1}(D_1,T')=\{(t,u^{-1}(D_1(p(t)))),(q,u^{-1}(D_1(p(q'))))\}$ = $\{(t,u^{-1}(D_1(t'))),(q,u^{-1}(D_1(q')))\}=\{(t,u^{-1}(\{2,4\})),(q,u^{-1}(\{2,4\}))\}=\{(t,\{11,15\}),(q,\{11,15\})\}$ is not an sOs in X.

f_{pu} is not $sTCONm$ since (D_1,T') is an sOs in Y but $f_{pu}^{-1}(D_1,T')$ is not a $sCOs$ in X.

f_{pu} is not $sSTRCONm$ since (D_1,T') is an sS in Y but $f_{pu}^{-1}(D_1,T')$ is not a $sCOs$ in X.

f_{pu} is $sSCONm$.

Main Results

Definition 7. A mapping $f_{pu}: SS(X_T) \to SS(Y_{T'})$, with $p: T \to T'$ and $u: X \to Y$, is called soft semi-totally continuous mapping written as $(sSTCONm)$if, $f_{pu}^{-1}(D,T') \tilde{\in} sCOs(X_T)\ \forall (D,T') \tilde{\in} sSOs(Y_{T'})$.

Example 2. Let $X=Y=\{0,1,2\}$, $T=\{t,q\}$ and $T'=\{t',q'\}$, "$\tau=\{\phi_T, X_T, (W_1,T), (W_2,T)\}$" and $\rho=\{\phi_{T'}, Y_{T'}, (D_1,T')\}$, where "$(W_1,T)=\{(t,\{0\}),(q,\{0\})\}$", $(W_2,T)=\{(t,\{1,2\}),(q,\{1,2\})\}$,

$(D_1,T')=\{(t',\{0\}),(q',\{0\})\}$. Describe the mapping $f_{pu}:SS(X_T)\to SS(Y_{T'})$ at this time, where $p:T\to T'$ and $u:X\to Y$ are distinguished by $p(t)=t'$, $p(q)=q'$, $u(1)=u(2)=0$, $u(0)=2$.

$sSOs(Y_{T'})=\{\emptyset_{T'}, Y_{T'}, (D_1,T'),\text{"}\{(t',\{0,1\}),(q',\{0,1\})\}\text{"},\{(t',\{0,2\}),(q',\{0,2\})\}\}$. Plainly, f_{pu} is a $sSTCONm$.

Proposition 1. Each sOs is $sSOs$.[5]

Proposition 2. Each $sSOs$ is a $sIOs$.[5]

Theorem 1. A soft mapping $f_{pu}:SS(X_T)\to SS(Y_{T'})$ is $(sSTCONm)$ if and only if $f_{pu}^{-1}(D,T')\tilde{\in}$ $sCOs(X_T)$ $\forall(D,T')\tilde{\in}\, sSCs(Y_{T'})$.

Proof. Assume that (W,T) is $sSCs$ in Y. Then $Y\tilde{\setminus}(W,T)$ is $sSOs$ in Y. We get, $f_{pu}^{-1}(Y\tilde{\setminus}(W,T))$ is $sCOs$ in . That is, $X\tilde{\setminus}f_{pu}^{-1}(W,T)$ is $sCOs$ in X. We have, $f_{pu}^{-1}(W,T)$ is $sCOs$ in .

Conversely, if (W,T) is $sSOs$ in Y, then, $Y\tilde{\setminus}(W,T)$ is $sSCs$ in Y. By hypothesis, $f_{pu}^{-1}(Y\tilde{\setminus}(W,T))=X\tilde{\setminus}f_{pu}^{-1}(W,T)$ is $sCOs$ in X, then, $f_{pu}^{-1}(W,T)$ is $sCOs$ in X. Henceforth, f_{pu} is $(sSTCONm)$.

Theorem 2. Each $(sSTCONm)$ is $(sTCONm)$.

Proof. Suppose that $f_{pu}:SS(X_T)\to SS(Y_{T'})$ is $(sSTCONm)$ and (U,T) is (sOs) in . By (Proposition 1) we get, (U,T) is $(sSOs)$ in Y. By suppose we have, $f_{pu}^{-1}(U,T)$ is $(sCOs)$ in X. Henceforth, f_{pu} is $(sTCONm)$.

Example 3. Let $X=Y=\{0,1,2\}$, $T=\{t,q\}$ and $T'=\{t',q'\}$, $\tau=\{\phi_T,X_T,(W_1,T),(W_2,T)\}$ and $\rho=\{\phi_{T'},Y_{T'},(D_1,T')\}$, where, $(W_1,T)=\{(t,\{0\}),(q,\{0\})\}$, $(W_2,T)=\{(t,\{1,2\}),(q,\{1,2\})\}$, $(D_1,T')=\{(t',\{0\}),(q',\{0\})\}$. Describe the mapping $f_{pu}:SS(X_T)\to SS(Y_{T'})$ at this time, where $p:T\to T'$ and $u:X\to Y$ are distinguished by $p(t)=t'$, $p(q)=q'$, $u(0)=0, u(1)=1, u(2)=2$.

$sSOs(Y_{T'})=\{\emptyset_{T'}, Y_{T'}, (D_1,T'),\{(t',\{0,1\}),(q',\{0,1\})\},\{(t',\{0,2\}),(q',\{0,2\})\}\}$. Plainly, f_{pu} is a $sTCONm$. But it isn't $sSTCONm$, because, $(W,T)=\{(t,\{0,1\}),(q,\{0,1\})\}$ is $sSOs$ in Y, but, $f_{pu}^{-1}(W,T)=(W,T)$ is not $sCOs$ in X.

Theorem 3. Each $(sSTRCONm)$ is $(sSTCONm)$.

Proof. Assume that $f_{pu}:SS(X_T)\to SS(Y_{T'})$ is $(sSTRCONm)$ and (U,T) is $(sSOs)$ in .,We get, $f_{pu}^{-1}(U,T)$ is $(sCOs)$ in X "(by assumption)". Henceforth, f_{pu} is $(sSTCONm)$.

Example 4. Let $X=Y=\{0,1,2\}$, $T=\{t,q\}$ and $T'=\{t',q'\}$, $\tau=\{\phi_T,X_T,(W_1,T),(W_2,T),(W_3,T),(W_4,T)\}$ and $\rho=\{\phi_{T'},Y_{T'},(D_1,T'),(D_2,T')\}$, where

$(W_1,T)=\{(t,\{0\}),(q,\{0\})\}$, $(W_2,T)=\{(t,\{1\}),(q,\{1\})\}$,

$(W_3,T)=\{(t,\{0,1\}),(q,\{0,1\})\}$, $(W_4,T)=\{(t,\{0,2\}),(q,\{0,2\})\}$,

$(D_1,T')=\{(t',\{0\}),(q',\{0\})\}$, $(D_2,T')=\{(t',\{1,2\}),(q',\{1,2\})\}$.

Describe the mapping $f_{pu}:SS(X_T)\to SS(Y_{T'})$ at this time, where $p:T\to T'$ and $u:X\to Y$ are distinguished by $p(t)=t'$, $p(q)=q'$, $u(0)=1, u(1)=0, u(2)=2$.

$sSOs(Y_{T'})=\{\emptyset_{T'}, Y_{T'}, (D_1,T'), (D_2,T')\}$. Plainly, f_{pu} is a $sSTCONm$. But it isn't $sSTRCONm$, because, for $(W,T)=\{(t',\{1\}),(q',\{1\})\}$ in Y, but, $f_{pu}^{-1}(W,T)=\{(t,\{0\}),(q,\{0\})\}$ is not $sCOs$ in X.

Theorem 4. Every $(sSTCONm)$ is $(sTSCONm)$.

Proof. Let $f_{pu}:SS(X_T)\to SS(Y_{T'})$ be $(sSTCONm)$ and (M,T) is any (sOs) in Y. By (Proposition 1) and by assumption, we get, $f_{pu}^{-1}(M,T)$ is $(sCOs)$ and then it is $(sSCOs)$ in X. . Henceforth, f is $(sTSCONm)$.

Example 5. Let $X = Y = \{0,1,2\}$, , $T = \{t,q\}$ and $T' = \{t',q'\}$ $\tau = \{\phi_T, X_T, (W_1,T), (W_2,T), (W_3,T)\}$ and $\rho = \{\phi_{T'}, Y_{T'}, (D_1,T')\}$,where, $(W_1,T) = \{(t,\{0\}),(q,\{0\})\}, (W_2,T) = \{(t,\{1\}),(q,\{1\})\}$,
$(W_3,T) = \{(t,\{0,1\}),(q,\{0,1\})\}$,"$(D_1,T') = \{(t',\{0\}),(q',\{0\})\}$",

Describe the mapping $f_{pu} : SS(X_T) \to SS(Y_{T'})$ at this time, where $p : T \to T'$ and $u : X \to Y$ are distinguished by $p(t) = t'$, $p(q) = q'$, $u(0) = 0, u(1) = u(2) = 1.$

$sSOs(Y_{T'}) = \{ \emptyset_{T'}, Y_{T'}, (D_1,T'), \{(t',\{0,1\}),(q',\{0,1\})\}, \{(t',\{0,2\}),(q',\{0,2\})\} \}$. Plainly, f_{pu} is a *sTSCONm* . But it isn't *sSTCONm* , because, for *sSOs* $(W,T) = \{(t',\{0\}),(q',\{0\})\}$ in Y, but, $f_{pu}^{-1}(W,T) = \{(t,\{0\}),(q,\{0\})\}$ is not *sCOs* in X.

Theorem 5. Every *(sSTCONm)* is *(sSCONm)*.

Proof. Let $f_{pu} : SS(X_T) \to SS(Y_{T'})$ be *(sSTCONm)* and *(M,T)* is any *(sOs)* in *Y*. By assumption, we get, $f_{pu}^{-1}(M,T)$ is *(sCOs)* and then it is *(sSCOs)* in *X*. Then, $f_{pu}^{-1}(M,T)$ is *(sSOs)* in *X*. Henceforth, *f* is *(sSCONm)*.

Example 6. Let $X = Y = \{0,1,2\}$, , $T = \{t,q\}$ and $T' = \{t',q'\}$, $\tau = \{\phi_T, X_T, (W_1,T)\}$ and $\rho = \{\phi_{T'}, Y_{T'}, (D_1,T'), (D_2,T')\}$,where, $(W_1,T) = \{(t,\{0\}),(q,\{0\})\}, (D_1,T') = \{(t',\{0\}),(q',\{0\})\}, (D_2,T') = \{(t',\{0,1\}),(q',\{0,1\})\}$.

Describe the mapping $f_{pu} : SS(X_T) \to SS(Y_{T'})$ at this time, where $p : T \to T'$ and $u : X \to Y$ are distinguished by $p(t) = t'$, $p(q) = q'$, $u(0) = 0, u(1) = 1, u(2) = 2.$

$sSOs(Y_{T'}) = \{ \emptyset_{T'}, Y_{T'}, (D_1,T'), (D_2,T'), \{(t',\{0,2\}),(q',\{0,2\})\}\}$. Plainly, f_{pu} is a *sSCONm* . But it isn't *sSTCONm* , because, for $(W,T) = \{(t',\{0\}),(q',\{0\})\}$ in Y, but, $f_{pu}^{-1}(W,T) = \{(t',\{0\}),(q',\{0\})\}$ is not *sCOs* in X.

As a result, there is the following connection:
"Soft Strong continuity" ⟹ "Soft semi-totally continuity" ⟹ "Soft totally continuity" ⟹ "Soft total semi-continuity" ⟹ "Soft semi-continuity".
In general, The contrary is not true.

Theorem 6. Let $f_{pu} : SS(X_T) \to SS(Y_{T'})$ be a function with *sTs* values for *X* and *Y*. Consequently, the following arguments are equivalent:

(i) f_{pu} is *(sSTCONm)*.

(ii) for every $x \in X$ and each, *(sSOs)* *(M,T)* in *Y* with $f_{pu}(x,T) \tilde{\in} (M,T) \forall t \in T$, there is a *sCOs* (U,T) in *X* s.t $x \tilde{\in} (U,T)$ and $f_{pu}(U,T) \tilde{\subseteq} (M,T)$.

Proof, (i) ⇒ (ii): Let f_{pu} be *(sSTCONm)* and (M,T) be any *(sSOs)* in *Y* containing $f_{pu}(x,T)$ so that $x \tilde{\in} f_{pu}^{-1}(M,T)$. $f_{pu}^{-1}(M,T)$ is *sCOs* in *X* "(by assumption)". Let $(U,T) = f_{pu}^{-1}(M,T)$ then (U,T) is *sCOs* in *X* and $x \tilde{\in} f(U,T)$. Also $f_{pu}(U,T) = f_{pu}(f_{pu}^{-1}(M,T)) \tilde{\subseteq} (M,T)$. Henceforth, $f_{pu}(U,T) \tilde{\subseteq} (M,T)$

(ii) ⇒ (i): Let (M,T) be *(sSOs)* in *Y* Let $x \tilde{\in} f_{pu}^{-1}(M,T)$ be any arbitrary point, we get, $f_{pu}(x,T) \tilde{\in} (M,T) \forall t \in T$. From (ii), we have, *sCOs* , $f_{pu}(D,T) \tilde{\subseteq} X$, containing x s.t. $f_{pu}(D,T) \tilde{\subseteq} (M,T)$, we conclude that $(D,T) = f_{pu}^{-1}(M,T)$, $x \tilde{\in} (D,T) \tilde{\subseteq} f_{pu}^{-1}(M,T)$, then, $f_{pu}^{-1}(M,T)$ is clopen neighborhood of x, because x is arbitrary, we get, $f_{pu}^{-1}(M,T)$ is clopen neighborhood every one of its points. Henceforth, it is *sCOs* in *X*. Thus, f_{pu} is *(sSTCONm)*.

Theorem 7. The composition two *(sSTCONm)* is *(sSTCONm)*.

Proof. Let $f_{pu} : SS(X_T) \to SS(Y_{T'})$, $u : X \to Y, p : T \to T'$ and $g_{p'u'} : SS(Y_{T'}) \to SS(Z_{T''})$, $u' : Y \to Z, p' : T' \to T''$ be any two *(sSTCONm)*. Let (M,T'') be *sSOs* in Z. Since g is *(sSTCONm)*, we get, $g_{p'u'}^{-1}(M,T'')$ is *sCOs* in Y, so it is *sOs* in Y.

By (Proposition 1), we get, $g^{-1}_{p'u'}(M,T'')$ is $(sSOs)$ in Y, since f_{pu} is $(sSTCONm)$, we get, $f^{-1}_{pu}(g^{-1}_{p'u'}(M,T''))=(f\circ g)^{-1}(M,T'')$ is $sCOs$ in X. Hence $g\circ f: SS(X_T)\to SS(Z_{T''})$ is $(sSTCONm)$.

Theorem 8. If $f_{pu}: SS(X_T)\to SS(Y_{T'})$ is $(sSTCONm)$ and $g_{p'u'}: SS(Y_{T'})\to SS(Z_{T'})$ is $(sIREm)$, then $g\circ f: SS(X_T)\to SS(Z_{T''})$ is $(sSTCONm)$.

Proof. Assume that $f_{pu}: SS(X_T)\to SS(Y_{T'})$ be $(sSTCONm)$ and $g_{p'u'}: SS(Y_{T'})\to SS(Z_{T'})$ be $(sIREm)$. Let (M,T'') be $sSOs$ in . $g^{-1}_{p'u'}(M,T'')$ is $sSOs$ in Y "(by assumption)". Also, $f^{-1}_{pu}(g^{-1}_{p'u'}(M,T''))=(f\circ g)^{-1}(M,T'')$ is $sCOs$ in X. Henceforth, $g\circ f$ is $(sSTCONm)$.

Theorem 9. If $f_{pu}: SS(X_T)\to SS(Y_{T'})$ is $(sSTCONm)$ and $g_{p'u'}: SS(Y_{T'})\to SS(Z_{T''})$ is $(sSCONm)$, then $g\circ f: SS(X_T)\to SS(Z_{T''})$ is $(sTCONm)$.

Proof. Let (M,T'') be sOs in Z. Since g is soft semi-continuous, we get, $g^{-1}_{p'u'}(M,T'')$ is $sSOs$ in Y "(by assumption)". Also, $f^{-1}_{pu}(g^{-1}_{p'u'}(M,T''))=(f\circ g)^{-1}(M,T'')$ is $sCOs$ in X. Henceforth, $g\circ f: SS(X_T)\to SS(Z_{T''})$ is $(sTCONm)$.

Theorem 10. Let $f_{pu}: SS(X_T)\to SS(Y_{T'})$ be $(sSTCONm)$ and $g_{p'u'}: SS(Y_{T'})\to SS(Z_{T''})$ be any mapping. Then, $g\circ f: SS(X_{T'})\to SS(Z_{T''})$ is $(sSTCONm)$ *iff* $g_{p'u'}$ is $(sIREm)$.

Proof. Suppose that $g_{p'u'}: SS(Y_{T'})\to SS(Z_{T''})$ be $(sIREm)$. Then the proof is complete(Theorem 8)

On the other wise, let $g\circ f$ be $(sSTCONm)$, let (M,T'') be $sSOs$ in Z. We get, $f^{-1}_{pu}(g^{-1}_{p'u'}(M,T''))=(g\circ f)^{-1}(M,T'')$ is $sCOs$ in X. Also, $g^{-1}_{p'u'}(M,T'')$ is $sSOs$ in Y. Henceforth, $g_{p'u'}$ is $(sIREm)$.

Definition 8. A soft mapping $f_{pu}: SS(X_T)\to SS(Y_{T'})$ is said to be soft semi-totally open written as $(sSTOm)$ if $f_{pu}(M,T)\tilde{\in}\ sCOs(Y_{T'})\ \forall(M,T)\tilde{\in}\ sSOs(X_T)$.

Example 7. Let $X=\{7,11,17\}$, $Y=\{2,4,6\}$, $T=\{t,q\}$, $T'=\{t',q'\}$

$\tau=\{\phi_T, X_T,(W_1,T)\}$, $\rho=\{\phi_{T'}, Y_{T'},(D_1,T'),(D_2,T'),(D_3,T')\}$,

Where, $(W_1,T)=\{(t,\{7,11\}),(q,\{7,11\})\}$,

$(D_1,T')=\{(t',\{2\}),(q',\{2\})\}$, $(D_2,T')=\{(t',\{4\}),(q',\{4\})\}$, $(D_3,T')=\{(t',\{2,4\}),(q',\{2,4\})\}$.

Describe the mapping $f_{pu}: SS(X_T)\to SS(Y_{T'})$ at this time, where $p: T\to T'$ and $u: X\to Y$ are distinguished by: $p(t)=t'$,

$p(q)=q'$, $u(17)=4$, $u(11)=6$, $u(7)=2$ $sOs(X_T)=\{\phi_T, X_T,(W_1,T)\}$

$\{\phi_T, X_T,(W_1,T),\{(t_1,\{7\}),(t_2,\{7\})\},\{(t_1,\{11\}),(t_2,\{11\})\},\{(t_1,\{7,17\}),(t_2,\{7,17\})\},$

$,\{(t_1,\{11,17\}),(t_2,\{11,17\})\}\}\ \tilde{\in}\ sSOs\ (X_T)$,

$sCOs(Y_{T'})=\{\phi_{T'}, Y_{T'}\}$

f_{pu} is not a $sSTOm$ because $f_{pu}(W_1,T)=\{(t',\{2,6\}),(q',\{2,6\})\}\ \tilde{\notin}\ sCOs\ (Y_{T'})$,

If $\rho=\{\phi_{T'}, Y_{T'},(D_1,T'),(D_2,T'),(D_3,T'),(D_4,T'),(D_5,T'),(D_6,T')\}$,

Where, $(D_1,T')=\{(t',\{2\}),(q',\{2\})\}$, $(D_2,T')=\{(t',\{4\}),(q',\{4\})\}$,

$(D_3,T')=\{(t',\{6\}),(q',\{6\})\}$, $(D_4,T')=\{(t',\{2,4\}),(q',\{2,4\})\}$, $(D_5,T')=\{(t',\{2,6\}),(q',\{2,6\})\}$,

$(D_6,T')=\{(t',\{4,6\}),(q',\{4,6\})\}$, then, f_{pu} is $sSTOm$.

Theorem 11. If a bijective $f_{pu}:SS(X_T)\to SS(Y_{T'})$ is $(sSTOm)$, then the image of each $sSCs$ in X is $sCOs$ in Y.

Proof. Let (M,T) be $sSCs$ in . Then $X\tilde{\setminus}(M,T)$ is $sSOs$ in X. We get, $f_{pu}(X\tilde{\setminus}(M,T))=Y\tilde{\setminus}f_{pu}(M,T)$ is $sCOs$ in Y "(by assumption)". Henceforth, $f_{pu}(M,T)$ is $sCOs$ in Y.

Theorem 12. The composition of the two $(sSTOm)$ is $(sSTOm)$.

Proof. Assume that f_{pu} and $g\circ f:SS(X_T)\to SS(Z_{T''})$ are any $(sSTOm)$. Let (M,T) be $sSOs$ in . Consider $g\circ f(M,T)=g(f(M,T))$. Since f is $(sSTOm)$, $f_{pu}(M,T)$ is $sCOs$ in Y. Also, it is sOs in Y. We get, it is $sSOs$ in Y (Proposition 1). We get, $g_{p'u'}(f_{pu}(M,T))$ is $sCOs$ in Z "(by assumption)". Henceforth, $g\circ f$ is $(sSTOm)$.

Definition 9. A $f_{pu}:SS(X_T)\to SS(Y_{T'})$ soft mapping with $p:T\to T'$ and $u:X\to Y$ is known as:

(i) i-continuous written as $(sICONm)$ [6] if $f_{pu}^{-1}(D,T')\tilde{\in} sIOs(X_T)\ \forall(D,T')\tilde{\in} sOs(Y_{T'})$.

(ii) Totally i-continuous written as $(sTICONm)$ if $f_{pu}^{-1}(D,T')\tilde{\in} sICOs(X_T)\ \forall(D,T')\tilde{\in} sOs(Y_{T'})$.

(iii) Strongly i-continuous written as $(sSTRICONm)$ if $f_{pu}^{-1}(D,T')\tilde{\in} sICOs(X_T)\ \forall(D,T')\tilde{\subseteq} Y_{T'}$.

(iv) i-irresolute written as $(siIREm)$ if $f_{pu}^{-1}(D,T')\tilde{\in} sIOs(X_T)\ \forall(D,T')\tilde{\in} sIOs(Y_{T'})$.

(v) i-totally continuous written as $(sITCONm)$ if $f_{pu}^{-1}(D,T')\tilde{\in} sCOs(X_T)\ \forall(D,T')\tilde{\in} sIOs(Y_{T'})$.

Example 8. Let $X=\{11,13,15,17\}$, $Y=\{2,4\}$, $T=\{t,q\}$, $T'=\{t',q'\}$.

Obviously, $\tau=\{\phi_T,X_T,(\beta_1,T),(\beta_2,T),(\beta_3,T)\}$, $\rho=\{\phi_{T'},Y_{T'},(\mu_1,T'),(\mu_2,T')\}$ are sTs over X and Y individually. Where,

$(\beta_1,T)=\{(t,\{13\}),(q,\{13\})\}$, $(\beta_2,T)=\{(t,\{15,17\}),(q,\{15,17\})\}$,

$(\beta_3,T)=\{(t,\{13,15,17\}),(q,\{13,15,17\})\}$, $(\mu_1,T')=\{(t',\{2\}),(q',\{2\})\}$, $(\mu_2,T')=\{(t',\{4\}),(q',\{4\})\}$.

Describe the mapping $f_{pu}:SS(X_T)\to SS(Y_{T'})$ at this time, where $p:T\to T'$ and $u:X\to Y$ are distinguished by: $p(t)=t'$, $p(q)=q'$, $u(13)=4$, , $u(11)=u(15)=u(17)=2$.

$sOs(X_T)=\{\phi_T,X_T,(\beta_1,T),(\beta_2,T),(\beta_3,T)\}$

$sCs(X_T)=\{\phi_T,X_T,\{(t,\{11,15,17\}),(q,\{11,15,17\})\},\{(t,\{11,13\}),(q,\{11,13\})\},\{(t,\{13\}),(q,\{13\})\}\}$

$\{\phi_T,X_T,(\beta_1,T),(\beta_2,T),(\beta_3,T)\},\{(t,\{11,15,17\}),(q,\{11,15,17\})\}\tilde{\in} sIOs(X_T)$.

$\{\phi_T,X_T,(\beta_1,T),,\{(t,\{11,15,17\}),(q,\{11,15,17\})\}\tilde{\in} sICOs(X_T)$. $sOs(Y_{T'})=sIOs(Y_{T'})=\{\phi_{T'},Y_{T'},(\mu_1,T'),(\mu_2,T')\}$.

f_{pu} is a $sICONm$, $sTICONm$, $sSTRICONm$, $siIREm$, $sITCONm$,

Proposition 3. Each $(sSCONm)$ is $(sICONm)$.[6]

Theorem 13. Each $(sTSCONm)$ is $(sTICONm)$.[6]

Proof. Suppose that $f_{pu}:SS(X_T)\to SS(Y_{T'})$ is $(sTSCONm)$ and (U,T') is (sOs) in . By suppose we have, $f_{pu}^{-1}(U,T')$ is $(sSCOs)$ in X. Thus, $f_{pu}^{-1}(U,T')$ is $(sICOs)$ in X (Proposition 2). Henceforth, f_{pu} is $(sTICONm)$.

Theorem 14. Each $(sSTRSCONm)$ is $(sSTRICONm)$.

Proof. Suppose that $f_{pu} : SS(X_T) \to SS(Y_{T'})$ is $(sSTRSCONm)$ and (U,T') is any soft subset of . By suppose we have, $f_{pu}^{-1}(U,T')$ is $(sSCOs)$ in X. Thus, $f_{pu}^{-1}(U,T')$ is $(sICOs)$ in X (Proposition 2). Henceforth, f_{pu} is $(sSTRICONm)$.

Theorem 15. Each $(sSTCONm)$ is $(sITCONm)$.

Proof. Suppose that $f_{pu} : SS(X_T) \to SS(Y_{T'})$ is $(sSTCONm)$ and (U,T') is any $(sSOs)$ in Y. By suppose we have, $f_{pu}^{-1}(U,T')$ is $(sCOs)$ in X. Also, (U,T') is $(sIOs)$ in Y (Proposition 2). Henceforth, f_{pu} is $(sITCONm)$.

Conclusions: From above we concluded many important theorems as follows: each $(sSTCONm)$ is $(sTCONm)$, each $(sSTRSCONm)$ is $(sSTRICONm)$, each $(sTSCONm)$ is $(sTICONm)$, the composition of two $(sSTOm)$ is $(sSTOm)$, and the composition of two $(sSTCONm)$ is $(sSTCONm)$.

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Conflict of interest

The author has no conflict of interest.

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شبه الاستمرارية التامة الناعمة في الفضاءات التبولوجية الناعمة

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المستخلص

التطبيقات شبه المستمرة التامة الناعمة والتي عرفناها في هذه الدراسة في الفضاءات التبولوجية الناعمة هي من التطبيقات المستمرة التامة الناعمة، حيث قمنا بأثبات مجموعة من الافكار للتطبيقات المستمرة الناعمة من فضاء تبولوجي ناعم الى فضاء تبولوجي ناعم اخر. بالأدلة والبراهين والامثلة للتوضيح والشرح لكل ما ذكر. العلاقات بين هذه الاصناف وعدة اصناف اخرى من التطبيقات الناعمة تم دراستها. كذلك مميزات عدة اصناف من هذه الدوال تم التطرق اليها ودراستها ايضا. اضافة الى ذلك قمنا بتعريف التطبيقات المستمرة الناعمة بالاعتماد على المجاميع المفتوحة الناعمة من النمط–i واستنتاج مدى علاقتها الوثيقة بالتطبيقات المستمرة الناعمة الاخرى التي تعتمد على المجاميع شبه المفتوحة الناعمة. ففي هذا البحث برهننا بان كل تطبيق شبه مستمر تام ناعم يكون مستمر تام ناعم، كل تطبيق شبه مستمر ناعم بقوة يكون مستمر من النمط–i ناعم بقوة، كل تطبيق شبه مستمر تام ناعم يكون مستمر من النمط–i تام ناعم، تركيب تطبيقين شبه مفتوحين تامين ناعمين يكون تطبيق شبه مفتوح تام ناعم. واخيرا تركيب كل تطبيقين شبه مستمرين تامين ناعمين يكون تطبيق شبيه مستمر تام ناعم.