Simple Method for Determination of The Deformed Rotor Basis States

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الخلاصة

تم في هذه الدراسة عرض طريقة بسيطة لحساب الحالات الاساسية للنـــوى الدورانيـــة المـــشوهة SU(3). حيث تم حساب قيم الزخوم الزاوية لغاية N=7 و تم المقارنة مـــع القـــيم المحـــسوبة ســـابقا و اظهرت تطابقا تاما

ABSTRACT

In this paper a simple method is introduced to calculate the basis states for the deformed rotor nuclei SU(3). The values of angular momentum have been calculated up to N=7. A comparison with the previous available results shows quite good .

INTRODUCTION

A nuclear model was proposed, by Arima and Iachello [1,2,3], called the interacting boson model or IBM. The IBM invoked an algebraic and group-theoretical approach. The basic ideas of the IBM [2,3,4,5,6] is to assume that low-lying collective states in even-even nuclei can be described by a system of interacting s and d bosons carrying angular momentum 0 and 2 respectively.

Both, the s(l=0) and d(l=2) bosons of the IBM-1(the first version of IBM) have six components (substates) and therefore define a six-dimensional space. This leads to a description in terms of the unitary group in six dimensions, U(6). A consideration of different reductions of U(6) leads to three dynamical symmetries [3,7,8,9] known as U(5), SU(3) and O(6) which are related to the geometrical idea of the spherical vibrator, deformed rotor, and asymmetric (γ-soft) deformed rotor, respectively.

One can shows that the three subalgebras, I, II and III, are the only possible ones [10]. In fact, starting from U(6) one has considered U(5), SU(3) and O(6) represented by three chains:

$$U(6) \begin{cases} \nearrow U(5) \supset O(5) \supset O(3) \supset O(2) & (I) \\ \rightarrow SU(3) \supset O(3) \supset O(2) & (II) \\ \searrow O(6) \supset O(5) \supset O(3) \supset O(2) & (III) \end{cases} \quad (1)$$

In the present work a simple equations to determine the basis states of SU(3) have been proposed.

Theory

One of the main uses of group (or algebra) chains is to construct bases in which the Hamiltonian operator can be diagonalized. One can construct bases that transform as representations of the appropriate groups and label those states with the corresponding quantum numbers [10].

Tensor representations of Lie groups are labeled by a set of integer numbers $[\lambda 1, \lambda 2, \ldots\ldots\ldots, \lambda_n]$ with $\lambda 1 \geq \lambda 2 \geq \ldots\ldots \geq \lambda_{n-1} \geq \lambda_n$ which are some times displayed pictorially in a Young tableau [10]. , as follows:

$$[\lambda 1, \lambda 2, \ldots\ldots\ldots] = \overbrace{\square\square\square}^{\lambda 1} \quad \overbrace{\square\square}^{\lambda 2} \quad (2)$$

For the case of identical bosons, the wave function must be totally symmetric [10]. This means that the dealing here will be only with the totally symmetric representation of U(6), characterized by a single number N, the total boson number can be represented by:

$$[N] \equiv \overbrace{\square\square\ldots\ldots\ldots\ldots\square}^{\text{N times}} \quad (3)$$

The labels needed to classify the states in SU(3) chain are given by [7]:

$$\begin{array}{ll} U(6) & [N,0,0,0,0,0] \equiv [N] \\ SU(3) & (f_1, f_2) \\ O(3) & L \\ O(2) & M_L \end{array} \quad (4)$$

Where L is the angular momentum, M_L is the third component of L and the Young tableau (f_1, f_2) is related to (λ, μ) by :

$$\begin{array}{l} \lambda = f_1 - f_2 \\ \mu = f_2 \end{array} \quad (5)$$

The values of (λ,μ) contained in a symmetric representation [N] of U(6) are restricted to be positive integer and obey the following rules [11]

$$\mu = 0,2,4$$
$$\lambda = 2N - 6l - 2\mu \qquad with\, l = 0,1,.... \qquad (6)$$

Therefore, the doublets (λ,μ) contain the following values; for N=1 they amount to (2,0); for N=2 we have (4,0) and (0,2); for N=3 consequently (6,0), (2,2) and (0,0) and so forth. Further doublets can taken from fig.(1), and given by [7]:

$$\begin{aligned}(\lambda,\mu) = (2N,0) \oplus (2N-4,2) \oplus (2N-8,4) \oplus .... &\oplus \begin{Bmatrix}(0,N)\\(2,N-1)\end{Bmatrix} \begin{Bmatrix}N = even\\N = odd\end{Bmatrix}\\ \oplus (2N-6,0) \oplus (2N-10,2) \oplus .... &\oplus \begin{Bmatrix}(0,N-3)\\(2,N-4)\end{Bmatrix} \begin{Bmatrix}N-3 = even\\N-3 = odd\end{Bmatrix}\\ \oplus (2N-12,0) \oplus (2N-16,2) \oplus ...... &\oplus \begin{Bmatrix}(0,N-6)\\(2,N-7)\end{Bmatrix} \begin{Bmatrix}N-6 = even\\N-6 = odd\end{Bmatrix}\end{aligned} \qquad (7)$$

The step to go from SU(3) to O(3) is not fully decomposable and needs an additional quantum number to classify the states uniquely. The corresponding number is called K. The values of L contained in each representation (λ,μ) are then given by the following algorithm:

$$L = K, K+1, K+2, ..., K + \max\{\lambda,\mu\} \qquad (8)$$

where

$$K = integer = \min\{\lambda,\mu\}, \min\{\lambda,\mu\} - 2, ..., 1\, or\, 0 \qquad for\ (\min\{\lambda,\mu\} = odd\, or\, even) \qquad (9)$$

with the exception of $K = 0$ for which

$$L = \max\{\lambda,\mu\}, \max\{\lambda,\mu\} - 2, ..., 1 \qquad for\ (\max\{\lambda,\mu\} = odd\, or\, even) \qquad (10)$$

Elliott' s basis has the drawback of not being orthogonal. For this reason it is convenient to introduce a new basis labeled by the quantum number $\chi_1, \chi_2, ..., \chi_n$ with $\chi_1 < \chi_2 < ...... < \chi_n$ and defined by [12]:

$$|(\lambda,\mu),\chi_i,L,M_L\rangle\rangle = \sum_{j=1}^{i} \chi_{ij} |(\lambda,\mu),K_j,L,M_L\rangle \qquad (11)$$

Table(1) shows the classification scheme for SU(3) group chain that represented by Iachello and Arima [10] where the angular momentum are calculated for N=0 to N=4 and Fig(1) shows a typical spectrum for SU(3) symmetrical for N=6 [10]. Instead of the old treatment represented by equations 8,9 and 10, we propose a new method by representing the quantum numbers K and L as follows:

$$K = 0,2,4,......., \lambda \text{ or } \mu \text{ which is the lowest} \quad (12)$$

And L takes the following values:

a) for K=0 ;

$$L = 0,2,4,..........., \lambda + \mu \quad (13)$$

b) for $K \neq 0$;

$$L=K,K+1,K+2,........,L_{max} \quad (14)$$

Where $L_{max} = \lambda + \mu + 1 - K$

Table(2) shows the calculated values of the quantum number K and the angular momentum L for the boson number N=0 to N=7.

DISCUSSION AND CONCLUSION

In this work, a simple method in calculating the basis states for SU(3) symmetry has been suggested. This is because certain formulas to determine these states are so difficult that one cannot apply them easily to calculate all states.

The new equations suggested in this work introduces a simple method in calculating the basis states for this SU(3) limit applicable to any numbers of bosons. It also gives results in agreement with the previous calculations depending on equations 8,9 and 10, which cannot be realized with out the knowledge of group theory. Being the mean of the max{ λ, μ } not very clear and if it means as one things the maximum value for λ or μ, equation (8) does not give the correct values of L for the case $K = 2$ and $(\lambda, \mu) = (8,2)$, where L takes the values (L=2,3,4....,10) which is not fit the real value for the maximum value of L which takes the value 9 as shown in fig(1). Also equation (10) gives the following values of L for the case $K = 0$, and $(\lambda, \mu) = (8,2)$, (L=0,2,4,...,8), but for L=10 does not exist which is not true as shown in fig.(1). But when equations (12,13 and 14) suggested in this study are applied, all cases shown in table(2), for all values of N from N=0 to N=7, are true as in comparison with fig.(2) for the same boson number [11], and these equations are easily applicable in any other case.

The agreement was found excellent. In other word, we could say that this method can be considered as a powerful tool and simple to calculate the angular momentum for any number of bosons.

Table(1) Classification scheme for the SU(3) group chain

U(6)	SU(3)		O(3)
N	(λ,μ)	χ	L
0	(0,0)	0	0
1	(2,0)	0	2,0
2	(4,0)	0	4,2,0
	(0,2)	0	2,0
3	(6,0)	0	6,4,2,0
	(2,2)	0	4,2,0
		2	3,2
	(0,0)	0	0
4	(8,0)	0	8,6,4,2,0
	(4,2)	0	6,4,2,0
		2	5,4,3,2
	(0,4)	0	4,2,0
	(2,0)	0	2,0

Table(2) Classification scheme for the SU(3) group chain using new method

U(6)	SU(3)		O(3)
N	(λ,μ)	K	L
0	(0,0)	0	0
1	(2,0)	0	2,0
2	(4,0)	0	4,2,0
	(0,2)	0	2,0
3	(6,0)	0	6,4,2,0
	(2,2)	0	4,2,0
		2	3,2
	(0,0)	0	0
4	(8,0)	0	8,6,4,2,0
	(4,2)	0	6,4,2,0
		2	5,4,3,2
	(0,4)	0	4,2,0
	(2,0)	0	2,0
5	(10,0)	0	10,8,6,4,2,0
	(6,2)	0	8,6,4,2,0
		2	7,6,5,4,3,2
	(2,4)	0	6,4,2,0
		2	5,4,3,2
	(4,0)	0	4,2,0
	(0,2)	0	2,0
6	(12,0)	0	12,10,8,6,4,2,0
	(8,2)	0	10,8,6,4,2,0
		2	9,8,7,6,5,4,3,2
	(4,4)	0	8,6,4,2,0
		2	7,6,5,4,3,2
		4	5,4
	(0,6)	0	6,4,2,0
	(6,0)	0	6,4,2,0
	(2,2)	0	4,2,0
		2	3,2
	(0,0)	0	0

Table(2) Classification scheme for the SU(3) group chain using new method (continue)

U(6)	SU(3)		O(3)
N	(λ,μ))	K	L
7	(14,0)	0	14,12,10,….,0
	(10,2)	0	12,10,8,……,0
		2	11,10,9,….. ,2
	(6,4)	0	10,8,6,……..,0
		2	9,8,7,……….,2
		4	7,6,5,……….,4
	(2,6)	0	8,6,4,……….,0
		2	7,6,5,……….,2
	(8,0)	0	8,6,4,……….,0
	(4,2)	0	6,4,2,0
		2	5,4,3,2
	(0,4)	0	4,2,0
	(2,0)	0	2,0

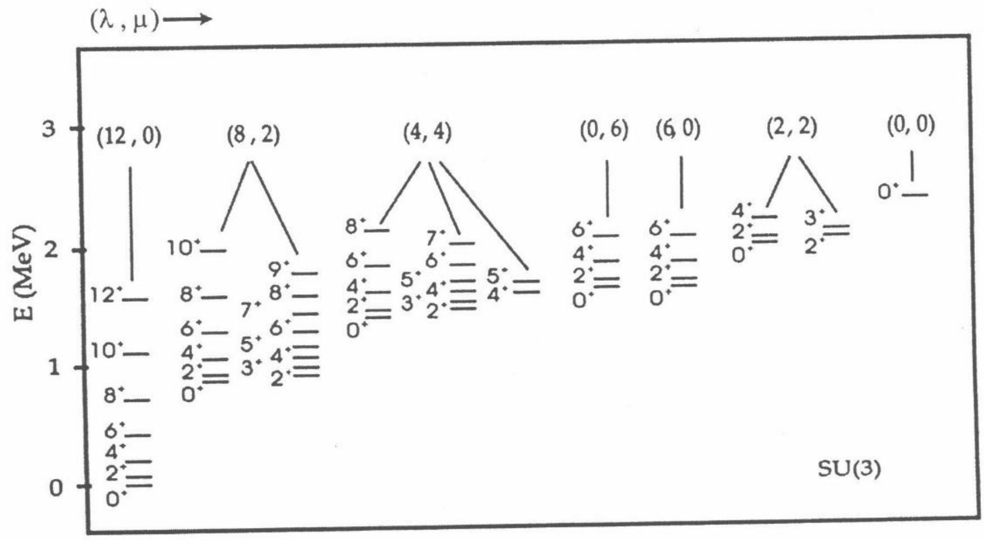

Figure (1) A typical spectrum with SU(3) symmetry and N=6 .In parentheses are the values of λ and μ . The angular momentum L of each state is shown to the left

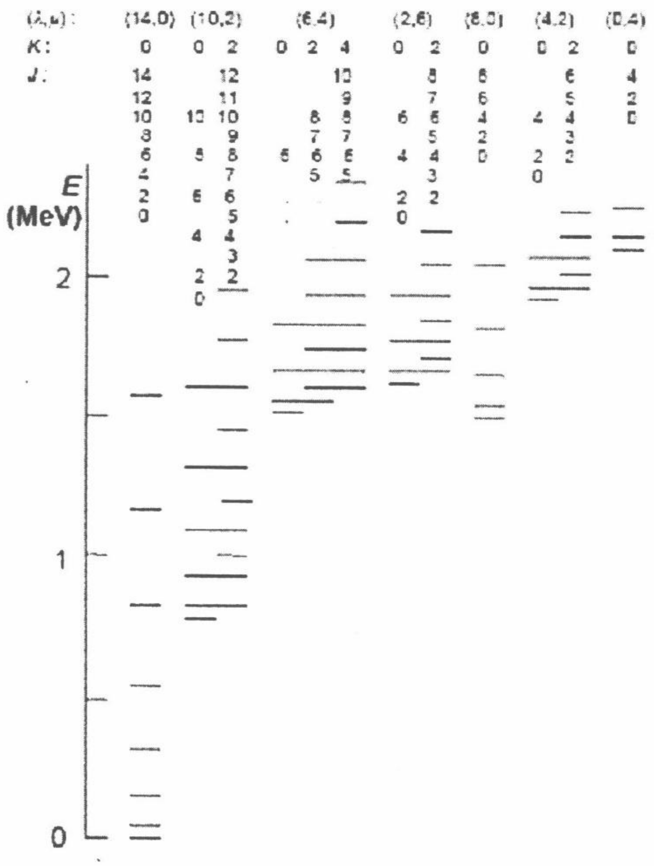

Figure (2) Typical spectrum of the SU(3) limit of the values N=7

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