Pθ-Topological Groups in Nonstandard Analysis

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ABSTRACT

The aim of this paper is to introduce and study a new class of topological groups called **Pθ-topological group**. By using some nonstandard techniques, we investigated some properties of Pθ-monads in Pθ-topological group.

Keywords: nonstandard analysis, topological group, monad..

الزمر التبولوجية من النمط Pθ في التحليل غير القياسي

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الملخص

الهدف من هذا البحث هو تقديم و دراسة صنف جديد من الزمر التبولوجية سميت بالزمر التبولوجية من النمط Pθ, باستخدام بعض أدوات التحليل غير القياسي، حصلنا على بعض النتائج لموناد من النمط -Pθ ، في الزمر التبولوجية من النمط -Pθ .

الكلمات المفتاحية: تحليل غير قياسي، زمر تبولوجية، هالة.

1- Introduction:-

In 1982, Mashhour A.S.*et al* [6], defined a new version of nearly open sets which is significant notion to the field of general topology called **preopen** sets .

There are several important concepts in topology in which can be defined in terms of preopen sets.

In 2000 Dontchev J.*et al*[3] introduced the concept of **pre -θ-open** sets , in this work we use the notion of pre -θ-open sets to define and study a new type of topological groups called **Pθ-topological group**, also we study some properties of Pθ-monads in Pθ-topological group. For this investigation, we need the following basic background in general topology and nonstandard analysis.

2- Basic Backgrounds in General Topology:-

Throughout this work, (X,τ) or (simply X) denotes a standard topological space on which no separation axioms are assumed unless explicitly stated, we recall the following definitions, notational conventions and characterizations.

The **closure** (resp., **interior**) of a subset A of a space X is denoted by **ClA**(resp. **IntA**).

Definition 2.1: A subset A of a space X is said to be

- **preopen** set [6] if and only if $A\subset$ IntClA.
- **β -open** set [2] if and only if $A\subset$ ClIntClA.
- **preclosed** set[2] if and only if $X\setminus A$ is preopen set. Equivalently, ClIntA$\subset$ A.
- **β-closed** set[2] if and only if $X\setminus A$ is β -open. Equivalently, IntClIntA$\subset$ A
- **θ-open** set[2] if for each $x\in A$, there is an open subset G of X such that $x\in G\subset ClG \subset A$
- **pre-θ-open** set[2] if for each $x\in A$, there is a preopen subset G of X such that $x\in G\subset pClG\subset A$.
- **sp-θ-open** set[2] if for each $x\in A$, there is a β -open subset G of X such that $x\in G \subset \beta ClG \subset A$.
- **pδ-open** set[4] if for each $x\in A$, there is a preopen subset G of X such that $x\in G \subset pIntpClG \subset A$.
- **θ-closed** if and only if $X\setminus A$ is θ-open set.
- **pθ-closed** if and only if $X\setminus A$ is pre-θ -open set.
- **sp-θ-closed** if and only if $X\setminus A$ is sp-θ -open set.
- **pδ-closed** if and only if $X\setminus A$ is pδ-open set.

The intersection of all pre-closed (resp., β -closed) sets containing A is called **pre-closure** (resp., **β -closure**) and denoted by **pClA**(resp., **βClA**).

The intersection of all θ-closed (resp.,pθ-closed,sp-θ-closed, and pδ-closed) sets containing A is called **θ-closure** (resp.,**pθ-closure**, **sp-θ-closure**, and **pδ-closure**) and denoted by $\mathbf{Cl_\theta A}$(resp., $\mathbf{pCl_\theta A}$, $\mathbf{spCl_\theta A}$, $\mathbf{pCl_\delta A}$).

The family of all pre-open(resp., β- open, θ-open,pre-θ-open, sp-θ-open, and pδ-open) sets of a space X is denoted by PO(X)(resp., βO(X), θO(X), PθO(X), SPθO(X), and PδO(X)).

The family of all pre-closed(resp., β- closed, θ- closed,pre-θ- closed, sp-θ- closed, and pδ- closed)sets of a space X is denoted by PC(X)(resp., βC(X), θC(X), PθC(X), SPθC(X), and PδC(X)).

Definition 2.2[4]: A topological space (X,τ) is called **p-regular**(resp., **p*-regular**) if and only if for each $x\in X$ and each closed (resp., preclosed)set F such that $x\notin F$, there exist two disjoint preopen sets A and B such that $x\in A$ and $F\subset B$.

Definition 2.3 [2]: A space X is **extremely disconnected** if the closure of every open set is an open set.

Theorem 2.4 [1]: Any union of pre-θ-open sets is a pre-θ-open set.

Theorem 2.5 [1]: For any space X, the following statements are true:

i) Every pre-θ-open set is preopen set.

ii) Every θ-open set is pre-θ-open set.

iii) Every pre-θ-open set is pδ-open set.

Theorem 2.6 [1]:

i) If X is extremely disconnected, then PθO(X)= PδO(X).
ii) If X is p –regular space , then $\tau \subset$ PθO(X).
iii) If X is p* –regular space, then PO(X)=PθO(X).

Proposition 2.7 [1]: Let X_1 and X_2 be two topological spaces and $X = X_1 \times X_2$ be the topological product,
let $A_i \in P\theta O(X_i)$ for $i=1,2$ then $A_1 \times A_2 \in P\theta O(X)$.

Definition 2.8: A mapping $f: (X, \tau) \to (Y, \rho)$ is said to be
i) **Pδ-irresolute** [4], if $f^{-1}(G) \in P\delta O(X, \tau)$, for each $G \in P\delta O(Y, \rho)$.
ii) **Pδ**-continuous** [4], if $f^{-1}(G) \in \tau$ for each $G \in P\delta O(Y, \rho)$.
iii) **completely preirresolute** [2], if $f^{-1}(G) \in PO(X, \tau)$, for each $G \in PO(Y, \rho)$.
iv) **faintly precontinuous** [2],if $f^{-1}(G) \in PO(X, \tau)$, for each $G \in \theta O(Y, \rho)$.
v) **strongly faintly precontinuous** [2], if $f^{-1}(G) \in \theta O(X, \tau)$, for each $G \in PO(Y, \rho)$.

Note that to define a Pθ-topological group, we introduce the following new type of continuity in topological spaces called **pθ-irresolute function**, some characterizations and relations are obtained for this definition.

Definition 2.9:A mapping $f: (X, \tau) \to (Y, \rho)$ is said to be **Pθ-irresolute** at a point x$\in$ X, if for each pre-θ-open set V of Y containing f(x), there exists a pre-θ-open set U of X such that $f(U) \subseteq V$.
If f is Pθ-irresolute at every point x$\in$ X, then it is called **Pθ-irresolute**.

Theorem 2.10: For any mapping $f: (X, \tau) \to (Y, \rho)$, the following statements are equivalent
i) *f* is Pθ-irresolute.
ii) The inverse image of every pre-θ-open set in Y is pre-θ-open set in X.
iii) The inverse image of every pre-θ-closed set in Y is pre-θ-closed set in X.
iv) $f(pCl_\theta(A)) \subset pCl_\theta(f(A))$, for each subset A of X.
v) $pCl_\theta(f^{-1}(B)) \subset f^{-1}(pCl_\theta(B))$, for each subset B of Y.
vi) $f^{-1}(pInt_\theta(B)) \subset pInt_\theta(f^{-1}(B))$, for each subset B of Y.
Proof: Straightforward.

Theorem 2.11: If X is extremely disconnected space, then every pδ-irresolute mapping is pθ-irresolute.
Proof: Let $f: (X, \tau) \to (Y, \rho)$ be a pδ-irresolute mapping, and let $G \in P\theta O(Y)$. Then, by Theorem 2.5(iii) we have $G \in P\delta O(Y)$, since *f* is pδ-irresolute function, then $f^{-1}(G) \in P\delta O(X)$.
Since X is extremely disconnected space, by Theorem 2.6(i), we get
$f^{-1}(G) \in P\theta O(X)$.
Hence *f* is pθ-irresolute.

Theorem 2.12: If X is p-regular space, then every pδ**-continuous mapping is pθ-irresolute mapping.
Proof: The proof is similar to Theorem 2.11

Theorem 2.13: If X is p*-regular space, then every completely preirresolute mapping is pθ-irresolute mapping.
Proof: It follows directly from Theorem 2.6(iii) and their definitions.

Theorem 2.14: For any mapping $f:(X,\tau) \to (Y,\rho)$, the following statements are true
i) Every pθ-irresolute is faintly precontinuous function.
ii) Every strongly faintly precontinuous is pθ-irresolute.
Proof: The proof is easy, and therefore is omitted.

Theorem 2.15: Let $f:X \to Y$ and $g:Y \to Z$ be two pθ-irresolute functions. Then the composition mapping $gof:X \to Z$ is pθ-irresolute.
Proof: The proof is obvious.

Theorem 2.16: If a mappings $f_i:X_i \to Y_i$ are pθ-irresolute for i=1,2. Then, the mapping $h:X_1 \times X_2 \to Y_1 \times Y_2$ defined by $h(x_1,x_2)=(f_1(x_1),f_2(x_2))$ is also pθ-irresolute.
Proof: Let $G_1 \times G_2 \subseteq Y_1 \times Y_2$, where $G_1\, and\, G_2$ are $pre-\theta$-open sets in $Y_1\, and\, Y_2$ respectively, by Proposition 2.7 we have $G_1 \times G_2 \in P\theta O(Y_1 \times Y_2)$.
Since f_1 and f_2 are pθ-irresolute mappings, then $f_1^{-1}(G) \in P\theta O(X_1)$ and
$f_2^{-1}(G) \in P\theta O(X_2)$ and $h^{-1}(G_1 \times G_2) = f_1^{-1}(G) \times f_2^{-1}(G) \in P\theta O(X_1 \times X_2)$.
Which implies that h is pθ-irresolute.

Theorem 2.17: Let X, Y_1, and Y_2 be any topological spaces, and $f_i: X \to Y_i$, for i=1,2 are mappings. If a function $g:X \to Y_1 \times Y_2$ defined by $g(x)=(f_1(x),f_2(x))$ is a pθ-irresolute then f_1 and f_2 are pθ-irresolute.
Proof: The proof is similar to Theorem 2.16.

3. Basic Backgrounds in Nonstandard analysis:

In this section, we use E. Nelson's Nonstandard Analysis construction, based on a theory called internal set theory IST[7] . The axioms of IST is the axiom of Zermelo-Frankel with the axiom of choice (briefly ZFC) together with three axioms which are the transfer principle (T), the idealization principle (I) , and the standardization principle (S), are stated by the following

Transfer principle

Let $A(x,t_1,t_2,\dots,t_n)$ be an internal formula with free variables $x,t_1,t_2,\dots,t_n$
Only then
$\forall^{st} t_1,t_2,\dots,t_n(\forall^{st} x\, A(x,t_1,t_2,\dots,t_n) \Longrightarrow (\forall x\, A(x,t_1,t_2,\dots,t_n)$

Example 3.1: Consider the following statement:
$\exists^{st}\, y \in R, \forall^{st} x \in R,\ \text{such that } x.y = x$
Applying transfer principle, we have $\exists\, y \in R, \forall x \in R,\ such\ that\ x.y = x$
Thus, we may assert that R has a unique multiplicative identity. Furthermore, recalling that we can identify R^{st} as a subset of R, we can say that this is 1.

The primary use of the transfer principle is that if one wishes to prove a theorem about the standard universe, it suffices to prove an analogous theorem with standard parameters in the enlarged universe.

Idealization Principle (I)

Let B(x,y) be an internal formula with free variables x ,y and with possibly other free variables then $\forall^{st\, fin} z \exists x \forall y \in Z \wedge B(x,y) \Leftrightarrow \exists x \forall^{st} y B(x,y)$.

Standardization Principle (S)

Let F(Z) be a formula, internal or external with free variables z and with possibly other free variables. Then,

$\forall^{st} x \exists^{st} y \forall^{st} z (z \in Y) \Leftrightarrow z \in X \wedge F(z).$
Every set or element defined in a classical mathematics is called standard.
Any set or formula which does not involve new predicates "standard, infinitesimal, limited, unlimited is called internal, otherwise it is called external

Definition 3.2 [5]: Let (X,τ) be a standard topological space. Then, the Pθ- monad at a standard point $a \in X$ is defined as follows
Pθ-monad = $\cap$\{pClA ; $A \in PO(X)$ and $a \in A$ \}, and is denoted by $\mu_{p\theta}(a)$.

Theorem 3.3[5]: Let (X,τ) be a standard topological space, and let a∈X be any element. Then, there exists a standard preopen H such that pClH $\subseteq \mu_{p\theta}(a)$.

Theorem 3.4[5]: Let A be a standard subset of a standard space X. Then, A is pre-θ-open set if and only if $\mu_{p\theta}(a) \subseteq A$. for each $a \in A$.
Proof: Assume that A is pre-θ-open set and let a∈A then there exists a standard pre-open G such that a∈G⊆ pClG ⊆A,
by transfer axiom pClG ⊆A for each G,A and a∈G∈ PO(X),
Now ∩\{pClG; G∈GP(a)\}⊆ pClG⊆A, by Definition 3.2, $\mu_{p\theta}(a) \subseteq A$.
Conversely, suppose that $\mu_{p\theta}(a) \subseteq A$ for each $a \in A$, then by Theorem 3.3, there exists a standard preopen set G such that pClG $\subseteq \mu_{p\theta}(a)$.
Thus a∈G⊂ pClG⊂A, for each standard a.
Therefore, by transfer principle, we have a∈G⊂ pClG⊂A, for each a.
Hence, A is pre-θ-open set.

4. Pθ-Topological Groups

Definition 4.1: Let G be a standard group and (G,τ) be a standard topological space. Then, (G,τ) is said to be **pθ-topological group**, if the mappings
$g: G \times G \to G$, defined by $g(x,y) = xy$ and
$h: G \to G$,, defined by $h(x) = x^{-1}$ are pθ-irresolute.

Example 4.2: Let $G=Z_2$ be a group of integer modulo 2, and τ be an indiscrete space then the maps
$g: G \times G \to G, g(x,y) = xy$ and $h: G \to G, h(x) = x^{-1}$ are pθ-irresolute.

Theorem 4.3: Let G be a standard group having a standard topology τ. Then (G,τ) is a standard pθ-topological group, if and only if the mapping
$f: G \times G \to G, \quad f(x,y) = xy^{-1}$ is pθ-irresolute.
Proof: The proof is obvious.

Theorem 4.4: (G,τ) is a standard pθ-topological group, if and only if the following conditions are satisfied

i) For every standard $x, y \in G$ and each standard pre-θ-open set H containing $x.y$, there exist a standard pre-θ-open sets U and V of x and y respectively such that $U.V \subseteq H$.

ii) For every standard $x \in G$ and each standard pre-θ-open set V containing x^{-1}, there exists a standard pre-θ-open sets U of x such that $U^{-1} \subseteq V$.
Where $U^{-1} = \{x^{-1}; x \in U\}$, $U.V = \{x.y; x \in U and\, y \in V\}$.
Proof: Let (G,τ) be a standard pθ-topological group,

and let H be a standard pre-θ-open set containing $x.y$, since the mapping $f: G \times G \to G$, defined by $f(x,y) = xy$ is pθ-irresolute, then .
$f^{-1}(H) = \{(x,y) \in G \times G; f(x,y) \in H\}$
$=\{(x,y) \in G \times G; x.y \in H\}$ is pre-θ-open subset of $G \times G$.
Thus, there exist pre-θ-open sets *U* and *V* of x and y, respectively in G such that $f^{-1}(H) = U \times V$
Now, $U.V = \{x.y; x \in U \text{ and } y \in V\} = \{x.y; (x,y) \in U \times V\}$
$= \{x.y; (x,y) \in f^{-1}(H)\}=\{x.y; f(x,y) \in H\}$
$= \{x.y; x.y \in H\} \subseteq H$
Let *V* be a pre-θ-open set containing x^{-1}, since the mapping $h: G \to G$, defined by $h(x) = x^{-1}$ is pθ-irresolute function, then $h^{-1}(V)$ is pre-θ-open set.
Therefore, there exists a pre-θ-open set U of *x* such that $h^{-1}(V) = U$.
Now $U^{-1} = \{x^{-1}, x \in U\}$
$= \{x^{-1}, x \in h^{-1}(V)\}$
$= \{x^{-1}, h(x) \in V\}$
$= \{x^{-1}, x^{-1} \in V\} \subseteq V$.
The converse part is obvious.

Definition 4.5: A mapping $f: (G,\tau) \to (G^*, \tau^*)$ is called pθ-homeomorphism if
i) *f* is bijective.
ii) *f* is pθ-irresolute.
iii) f^{-1} is pθ-irresolute.

Theorem 4.6: Let (G,τ) be a standard pθ-topological group, then the following mappings are pθ-homeomorphism.
i) $r_a: (G,\tau) \to (G,\tau)$, defined by $r_a(x) = xa$.
ii) $l_a: (G,\tau) \to (G,\tau)$ .defined by $l_a(x) = ax$.
iii) $f: (G,\tau) \to (G,\tau)$ .defined by $f(x) = x^{-1}$.
iv) $g: (G,\tau) \to (G,\tau)$. defined by $g(x) = axa^{-1}$.
are pθ-homeomorphism, for a fixed $a \in G$.
Proof: As a sample we proof (i)
It is clear that $r_a: (G,\tau) \to (G,\tau)$, defined by $r_a(x) = xa$ is bijective mapping.
Let H be a pre-θ-open set containing *x.a*. Since (G,τ) is a pθ-topological group , then by Theorem 4.4, there exists a pre-θ-open sets *U* and *V* of x and a respectively such that $U.V \subseteq H$.
Therefore $r_a(U) \subseteq H$.
Hence r_a is pθ-irresolute.
Now let $y = r_a(x)$, then $y = xa, x = ya^{-1}$
Which implies that that $r_a^{-1}(x) = r_{a^{-1}}(x) = xa^{-1}$
By similar way one can prove that r_a^{-1} is pθ-irresolute.

Theorem 4.7: Let (G,τ) be a pθ-topological group, and let U and V be a subset of G,g $\in$ G, then

i) If V is a pre-θ-open set, then Vg, gV, gVg^{-1} and V^{-1} are pre-θ-open sets.
ii) If U is a pre-θ-closed set, then Ug, gU, gUg^{-1} and V^{-1} are pre-θ-closed sets.
iii) If V is a pre-θ-open set and A is any subset of G, then VA and AV are pre-θ-open sets.

Proof: (i) and (ii) follow directly from Theorem 4.6 .
(iii) Since, $VA = \bigcup_{a\in A} Va$, by part (i),Va is a pre-θ-open set,
by Theorem(2.4) , we have VA is pre-θ-open set.
By similar way, we can prove that AV is pre-θ-open set.

Theorem 4.8:
A non-trivial standard pθ-topological group has no fixed point property.

Proof: Let (G, τ) be a standard pθ-topological group, for any $a \in G$ such that $a \neq e$, where e is the identity element in G.
The mapping $r_a: (G, \tau) \to (G, \tau)$, defined by $r_a(x) = xa$ is pθ-irresolute function
Suppose that $r_a(x) = x$, then $xa = x$, since $x \in G$ and G is a group,
Therefore, a=e, which is contradiction

5. Some properties of Pθ-monads in pθ-topological groups:

In this section, we give some properties of pθ-monads in pθ-topological groups, by using nonstandard techniques.

Theorem 5.1: Let a and b be any two standard points in pθ-topological group (G, τ), then $\mu_{p\theta}(a).\mu_{p\theta}(b) \subseteq \mu_{p\theta}(a.b)$.

Proof: Let $x \in \mu_{p\theta}(a)$ and $y \in \mu_{p\theta}(b)$. We have to show that for any pre-θ-open set W of $a.b$ in G $x.y \in W$, By Theorem 3.4 there exists two standard pre-θ-open sets U and V of a and b respectively, then $x \in U$ and $y \in V$.
Since (G, τ) is a pθ-topological group, by Theorem 4.4, for any standard pre-θ-open set W containing $a.b$, we have $U.V \subseteq W$, therefore $x.y \in W$.
Hence $\mu_{p\theta}(a).\mu_{p\theta}(b) \subseteq \mu_{p\theta}(a.b)$.

Theorem 5.2: Let a be a standard points in pθ-topological group (G, τ), then
$\mu_{p\theta}(a^{-1}) = (\mu_{p\theta}(a))^{-1}$.

Proof: Let V be a pre-θ-open set containing a^{-1}. Since, (G, τ) is a standard pθ-topological group, by Theorem 4.4 there exists a standard pre-θ-open sets U of x such that $a^{-1} \in U^{-1} \subseteq V$.
Then by Theorem 3.2 $\mu_{p\theta}(a^{-1}) \subseteq V$, $\mu_{p\theta}(a) \subseteq U$, and $U^{-1} \subseteq V$.
Therefore, $(\mu_{p\theta}(a))^{-1} \subseteq U^{-1} \subseteq V$, as V was an arbitrary standard pre-θ-open set, since PθO(X) ⊆ PO(X), and since $V \subseteq pClV$, we have
$(\mu_{p\theta}(a))^{-1} \subseteq \cap \{pClV \,;\, V \in PO(X, a^{-1})\} = \mu_{p\theta}(a^{-1})$.
If we replace a by a^{-1}, we get
$\mu_{p\theta}(a^{-1}) \subseteq (\mu_{p\theta}(a))^{-1}$.
Which implies that $\mu_{p\theta}(a^{-1}) = (\mu_{p\theta}(a))^{-1}$.

Theorem 5.3:
Let a and b be any two standard points in standard pθ-topological group (G, τ), then $\mu_{p\theta}(a).\mu_{p\theta}(b) = \mu_{p\theta}(a.b)$.

Proof: The proof is similar to Theorem 5.1.

Theorem 5.4:
If *U* is a standard pre-θ-open subset of *G*, then *U.a* is also standard pθ-open subset of *G*

Proof: Let U be a standard pre-θ-open subset of *G*, and let $b \in U$, then by Theorem 3.3, $\mu_{p\theta}(b) \subseteq U^{st}$, by transfer principle we have $\mu_{p\theta}(b) \subseteq U$,
Now, let $c \in U.a$, then *c*=*da*, for some $d \in U$
$\mu_{p\theta}(d).a \subseteq \mu_{p\theta}(d).\mu_{p\theta}(a)$ by Theorem 5.3, we have
$\mu_{p\theta}(d).a \subseteq \mu_{p\theta}(da) = \mu_{p\theta}(c)$ .....*(1)*
If $e \in \mu_{p\theta}(c)$, then $f = ea^{-1}$ such that
$f \in \mu_{p\theta}(c)\mu_{p\theta}(a^{-1}) = \mu_{p\theta}(c.a^{-1}) = \mu_{p\theta}(d)$.
Thus, $e = e.a^{-1}.a = f.a \in \mu_{p\theta}(d).a,$
that is $\mu_{p\theta}(c) \subseteq \mu_{p\theta}(d).a$ ……….(2)
from (1) and (2), we have $\mu_{p\theta}(c) = \mu_{p\theta}(d).a$
Hence, $\mu_{p\theta}(c) \subseteq U.a$, by Theorem 3.4 we obtain that *U.a* is pre-θ-open set.

Theorem 5.5: Let (G, τ) be a standard pθ-topological group, then *U* is a standard pre-θ-open subset of *G*, if and only if *U.a* is also standard pθ-open subset of *G* .
Proof: From Theorem 5.4, we have if *U* is a standard pre-θ-open subset of *G*, then *U.a* is also standard pre-θ-open subset of *G* .
It is enough to show that if *U.a* is a standard pre-θ-open subset of *G*, then *U* is also standard pre-θ-open subset of *G*, for any standard element *a* in G, since G is a group and $a \in G$, then $a^{-1} \in G$, by Theorem 5.4 $(U.a).a^{-1} = U$ is also pre-θ-open set.

Theorem 5.6: $\mu_{p\theta}(e)$ is a subgroup of a group *G***,** where *e* is the identity element in *G***.**
Proof: It is clear that $e \in \mu_{p\theta}(e)$, let $a, b \in \mu_{p\theta}(e)$,
Then, $a.b \in \mu_{p\theta}(e).\mu_{p\theta}(e)$, by Theorem 5.1, we have $a.b \in \mu_{p\theta}(e)$
Let $\in \mu_{p\theta}(e)$, then $a^{-1} \in (\mu_{p\theta}(e))^{-1}$, by Theorem 5.2, $a^{-1} \in \mu_{p\theta}(e)$
Hence, $\mu_{p\theta}(e)$ is a subgroup of *G*.

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