

## A Generalization of A Contra Pre Semi-Open Maps

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### ABSTRACT

The concept of  $\theta$ -semi-open sets in topological spaces was introduced in 1984 and 1986 by T. Noiri [9, 10]. In this paper we introduce and study a generalization of a contra pre semi-open maps due to (Caldas and Baker) [3], it is called contra pre  $\theta$ s-open maps, the maps whose images of a  $\theta$ -semi-open sets is  $\theta$ -semi-closed. Also, we introduce and study a new type of closed maps called contra pre  $\theta$ s-closed maps, which is stronger than contra pre semi-closed due to Caldas [2], the maps whose image of a  $\theta$ -semi-closed sets is  $\theta$ -semi-open. 1991 Math. Subject Classification: 54 C10, 54 D 10.

**Keywords:**  $\theta$ -semi-open sets, Contra pre  $\theta$ s-open and Contra pre  $\theta$ s-closed maps.

تعميم تعميم للدوال شبه مفتوحة من النمط  $\theta$  Pre contra

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### المخلص

عرف T. Noiri مفهوم مجموعة شبه مفتوحة من النمط  $\theta$  في الفضاء التوبولوجي في [9, 10] سنة 1984 و 1986. في هذا البحث نعرف و ندرس تعميم للدوال شبه مفتوحة من النمط  $\theta$  Pre contra المقدمة من قبل Caldas و Baker في [3] ، التي تسمى الدالة شبه المفتوحة من النمط  $\theta$  Pre contra . وهي الدوال التي تكون صور المجموعات شبه المفتوحة من النمط  $\theta$  ، شبه المغلقة من النمط  $\theta$  ، كما نعرف و ندرس نمطاً جديداً من الدوال المغلقة تسمى الدوال شبه مغلقة من النمط  $\theta$  Pre contra ، وهذه الدوال أقوى من الدوال شبه المغلقة من النمط  $\theta$  Pre contra المقدمة من قبل (caldas) في [2] ، وهي الدوال التي تكون صور المجموعات شبه المغلقة من النمط  $\theta$  ، شبه مفتوحة من النمط  $\theta$  .

الكلمات المفتاحية: مجموعة شبه مفتوحة من النمط  $\theta$ ، دوال شبه مفتوحة من النمط  $\theta$  Pre contra، دوال شبه مغلقة من النمط  $\theta$  Pre contra.

### 1. Introduction

The concept of  $\theta$ -semi-open set in topological spaces was introduced in 1984 and 1986 by T. Noiri [9, 10], which depends on semi-open sets due

to N. Levine [8]. When semi-open sets are replaced by  $\theta$ -semi-open sets, new results are obtained. M. Caldas and C. Baker defined and studied the concept of contra pre semi-open maps [3], where the maps whose images of semi-open sets are semi-closed.

In this direction we shall define the concept of Pre  $\theta$ s-open maps. In this paper we introduce two new types of open and closed maps called contra pre  $\theta$ s-open and contra pre  $\theta$ s-closed maps via the concept of  $\theta$ -semi-open sets and study some of their basic properties. We also establish relationships among these maps with other types of continuity, openness and closedness.

## 2. Preliminaries

Throughout the present paper, spaces always mean topological spaces on which no separation axiom is assumed unless explicitly stated. Let  $S$  be a subset of a space  $X$ . The closure and the interior of  $S$  are denoted by  $Cl(S)$  and  $Int(S)$ , respectively. A subset  $S$  is said to be regular open (resp. semi-open [8]) if  $S = Int(Cl(S))$  (resp.  $S \subset Cl(Int(S))$ ). A subset  $S$  is said to be  $\theta$ -semi-open [9] if for each  $x \in S$ , there exists a semi-open set  $U$  in  $X$  such that  $x \in U \subset Cl(U) \subset S$ . The complement of each regular open (resp. semi-open and  $\theta$ -semi-open) set is called regular closed (resp. semi-closed and  $\theta$ -semi-closed). The family of all semi-open (resp. semi-closed,  $\theta$ -semi-open and  $\theta$ -semi-closed) sets of  $X$  is denoted by  $SO(X)$  (resp.  $SC(X)$ ,  $\theta SO(X)$  and  $\theta SC(X)$ ). A point  $x$  is said to be in the  $\theta$ -semi-closure [10] of  $S$ , denoted by  $sCl_{\theta}(S)$ , if  $S \cap Cl(U) \neq \emptyset$  for each  $U \in SO(X)$  containing  $x$ . If  $S = sCl_{\theta}(S)$ , then  $S$  is called  $\theta$ -semi-closed. A point  $x$  is said to be in the  $\theta$ -semi-interior [10] of  $S$  denoted by  $sInt_{\theta}(S)$ , if  $Cl(U) \subset S$  for some  $U \in SO(X)$  containing  $x$ . If  $S = sInt_{\theta}(S)$ , then  $S$  is called  $\theta$ -semi-open. For each  $U \in SO(X)$ ,  $Cl(U)$  is  $\theta$ -semi-open and hence every regular closed set is  $\theta$ -semi-open. Therefore,  $x \in sCl_{\theta}(S)$  if and only if  $S \cap A \neq \emptyset$  for each  $\theta$ -semi-open set  $A$  containing  $x$ . A function  $f: X \rightarrow Y$  is said to be contra pre semi-open [3] (resp. contra pre semi-closed [2]) if for each semi-open (resp. semi-closed) set  $U$  of  $X$ ,  $f(U) \in SC(Y)$  (resp.  $f(U) \in SO(Y)$ ).

## 3. Contra pre $\theta$ s-open and contra pre $\theta$ s-closed maps

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a map from a topological space  $(X, \tau)$  into a topological space  $(Y, \sigma)$ .

**Definition 3.1:** A map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be contra pre  $\theta$ s-open (resp. contra pre  $\theta$ s-closed) if  $f(A)$  is  $\theta$ -semi-closed (resp.  $\theta$ -semi-open) in  $(Y, \sigma)$ , for each set  $A \in \theta SO(X, \tau)$  ( resp.  $A \in \theta SC(X, \tau)$ ).

The proof of the following two lemmas follows directly from their definitions and, therefore, they are omitted.

**Lemma 3.1:** Every contra pre semi-open map is contra pre  $\theta$ s-open.

**Lemma 3.2:** Every contra pre  $\theta$ s-closed map is contra pre semi-closed.

The converse of the above lemmas is not true in general as it is shown by the following examples.

**Example 3.1:** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ . Then the family of all semi-open subsets of  $X$  with respect to  $\tau$  is:

$SO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$  and the family of all  $\theta$ -semi-open subsets of  $X$  with respect to  $\tau$  is  $\theta SO(X) = \{\phi, X, \{b\}, \{a, c\}\}$ . The identity map

$f : (X, \tau) \rightarrow (X, \tau)$  is contra pre  $\theta$ s-open map, but it is not contra pre semi-open maps.

**Example 3.2:** Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ . Then, the family of all semi-open subsets of  $X$  with respect to  $\tau$  is:

$SO(X) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$  and the family of all  $\theta$ -semi-open subsets of  $X$  with respect to  $\tau$  is :

$\theta SO(X) = \{\phi, X, \{a, c\}, \{b, c\}\}$ . Define a function

$f : (X, \tau) \rightarrow (X, \tau)$  as follows:

$f(a) = b, f(b) = f(c) = a$ . Then  $f$  is contra pre semi-closed, but it is not contra pre  $\theta$ s-closed.

**Remark 3.1:** Contra pre  $\theta$ s-openness and contra pre  $\theta$ s-closedness are equivalent if the map is bijective.

**Theorem 3.1:** For a map  $f : X \rightarrow Y$  the following are equivalent:

i)  $f$  is contra pre  $\theta$ s-open;

ii) for every subset  $D$  of  $Y$  and for every  $\theta$ -semi-closed subset  $G$  of  $X$  with

$f^{-1}(D) \subset G$ , there exists a  $\theta$ -semi-open subset  $B$  of  $Y$  with  $D \subset B$  and

$f^{-1}(B) \subset G$ ;

iii) for every  $y \in Y$  and for every  $\theta$ -semi-closed subset  $G$  of  $X$  with

$f^{-1}(y) \subset G$ , there exists a  $\theta$ -semi-open subset  $B$  of  $Y$  with  $y \in B$  and

$f^{-1}(B) \subset G$ .

**Proof: (i)⇒(ii).** Let  $D$  be a subset of  $Y$  and let  $G$  be a  $\theta$ -semi-closed subset of  $X$  with  $f^{-1}(D) \subset G$ . Set,  $B = Y \setminus f(X \setminus G)$ . Since  $f$  is contra pre  $\theta$ -open, then  $B$  is a  $\theta$ -semi-open set of  $Y$  and since  $f^{-1}(D) \subset G$  we have  $f(X \setminus G) \subset Y \setminus D$  and hence  $D \subset B$ . Also,

$$f^{-1}(B) = X \setminus [f^{-1}(f(X \setminus G))] \subset X \setminus (X \setminus G) = G.$$

**(ii)⇒(iii).** It is sufficient, set  $D = \{y\}$ , we get the result.

**(iii)⇒(i).** Let  $A$  be a  $\theta$ -semi-open subset of  $X$  with  $y \in Y \setminus f(A)$  and let  $G = X \setminus A$ . By (iii), there exists a  $\theta$ -semi-open subset  $B_y$  of  $Y$  with  $y \in B_y$  and  $f^{-1}(B_y) \subset G$ . Then,  $y \in B_y \subset Y \setminus f(A)$ .

Hence  $Y \setminus f(A) = \cup \{B_y : y \in Y \setminus f(A)\}$ . Therefore, by [6, Lemma 2.2] that  $Y \setminus f(A)$  is  $\theta$ -semi-open. Thus,  $f(A)$  is a  $\theta$ -semi-closed subset in  $Y$ .

**Theorem 3.2:** For a map  $f : X \rightarrow Y$  the following are equivalent:

- i)  $f$  is contra pre  $\theta$ s-closed;
- ii) for every subset  $D$  of  $Y$  and for every  $\theta$ -semi-open subset  $A$  of  $X$  with  $f^{-1}(D) \subset A$ , there exists a  $\theta$ -semi-closed subset  $H$  of  $Y$  with  $D \subset H$  and  $f^{-1}(H) \subset A$ .

**Proof: (i)⇒(ii).** Let  $D$  be a subset of  $Y$  and let  $A$  be a  $\theta$ -semi-open subset of  $X$  with  $f^{-1}(D) \subset A$ . Set,  $H = Y \setminus f(X \setminus A)$ . Since  $f$  is contra pre  $\theta$ s-closed, therefore,  $H$  is a  $\theta$ -semi-closed set of  $Y$  and since  $f^{-1}(D) \subset A$ , we have  $f(X \setminus A) \subset X \setminus D$  and hence  $D \subset H$ . Also,  $f^{-1}(H) \subset A$ .

**(ii)⇒(i).** Let  $G$  be a  $\theta$ -semi-closed subset of  $X$ . Set,

$$D = Y \setminus f(G) \text{ and let } A = X \setminus G.$$

Hence  $f^{-1}(D) = f^{-1}(Y \setminus f(G)) = X \setminus f^{-1}(f(G)) \subset X \setminus G = A$ . By assumption, there exists a  $\theta$ -semi-closed set  $H \subset Y$  for which  $D \subset H$  and  $f^{-1}(H) \subset A$ . It follows that  $D = H$ . If  $y \in H$  and  $y \notin D$ , then  $y \in f(G)$ .

therefore,  $y = f(x)$  for some  $x \in G$  and we have  $x \in f^{-1}(H) \subset A = X \setminus G$  which is a contradiction. Since  $D = H$ , that is,  $Y \setminus f(G) = H$ , which implies that  $f(G)$  is  $\theta$ -semi-open and hence  $f$  is contra pre  $\theta$ s-closed.

Taking the set  $D$  in Theorem 3.2 to be  $\{y\}$  for  $y \in Y$  we obtain the following result.

**Corollary 3.1:** If  $f : X \rightarrow Y$  is contra pre  $\theta$ s-closed map, then for every  $y \in Y$  and every  $\theta$ -semi-open subset  $A$  of  $X$  with  $f^{-1}(y) \subset A$ , there exists a  $\theta$ -semi-closed subset  $H$  of  $Y$  with  $y \in H$  and  $f^{-1}(H) \subset A$ .

**Theorem 3.3:** A map  $f : X \rightarrow Y$  is contra pre  $\theta$ s-open if and only if for each  $x \in X$  and each semi-open set  $S$  in  $X$  containing  $x$ , there exists a  $\theta$ -semi-closed set  $H$  in  $Y$  containing  $f(x)$  such that  $H \subset f(Cl(S))$ .

**Corollary 3.2:** A map  $f : X \rightarrow Y$  is contra pre  $\theta$ s-open if and only if for each  $x \in X$  and each  $\theta$ -semi-open subset  $A$  of  $X$  containing  $x$ , there exists a  $\theta$ -semi-closed subset  $H$  of  $Y$  containing  $f(x)$  such that  $H \subset f(A)$ .

**Corollary 3.3:** A map  $f : X \rightarrow Y$  is contra pre  $\theta$ s-open, then for each  $x \in X$  and each regular closed subset  $R$  of  $X$  containing  $x$ , there exists a  $\theta$ -semi-closed subset  $H$  of  $Y$  containing  $f(x)$  such that  $H \subset f(R)$ .

**Theorem 3.4:** A map  $f : X \rightarrow Y$  is contra pre  $\theta$ s-closed if and only if for each  $x \in X$  and each  $\theta$ -semi-closed subset  $G$  of  $X$  containing  $x$ , there exists a semi-open subset  $W$  of  $Y$  containing  $f(x)$  such that  $Cl(W) \subset f(G)$ .

**Corollary 3.4:** A map  $f : X \rightarrow Y$  is contra pre  $\theta$ s-closed if and only if for each  $x \in X$  and each  $\theta$ -semi-closed subset  $G$  of  $X$  containing  $x$ , there exists a  $\theta$ -semi-open subset  $B$  of  $Y$  containing  $f(x)$  such that  $B \subset f(G)$ .

**Theorem 3.5:** For a map  $f : X \rightarrow Y$ , the following are equivalent:

- a)  $f$  is contra pre  $\theta$ s-open;
- b)  $f(sInt_{\theta}(A)) \subset sCl_{\theta}(f(A))$  for each subset  $A$  of  $X$ ;
- c)  $sInt_{\theta}(f^{-1}(B)) \subset f^{-1}(sCl_{\theta}(B))$  for each subset  $B$  of  $Y$ ;
- d)  $f^{-1}(sInt_{\theta}(B)) \subset sCl_{\theta}(f^{-1}(B))$  for each subset  $B$  of  $Y$ .

**Proof: (a)⇒(b).** Suppose  $f$  is contra pre  $\theta$ s-open maps and  $A \subset X$ . Since  $sInt_{\theta}(A) \subset A$ ,  $f(sInt_{\theta}(A)) \subset f(A)$  and hence  $f(sInt_{\theta}(A)) \subset sCl_{\theta}(f(A))$ .

**(b)⇒(c).** Let  $B$  be any subset of  $Y$ . Then  $f^{-1}(B) \subset X$ . Therefore, we apply (b), we obtain  $f(sInt_{\theta}(f^{-1}(B))) \subset sCl_{\theta}(f(f^{-1}(B))) \subset sCl_{\theta}(B)$ . Thus,  $sInt_{\theta}(f^{-1}(B)) \subset f^{-1}(sCl_{\theta}(B))$ .

**(c)⇒(d).** In (c), we take  $Y \setminus B$  instead of  $B$ , we get  $sInt_{\theta}(f^{-1}(Y \setminus B)) \subset f^{-1}(sCl_{\theta}(Y \setminus B))$ . Then,  $sInt_{\theta}(X \setminus f^{-1}(B)) \subset f^{-1}(Y \setminus sCl_{\theta}(B))$ , which implies that  $X \setminus sCl_{\theta}(f^{-1}(B)) \subset X \setminus f^{-1}(sInt_{\theta}(B))$ . Hence  $f^{-1}(sInt_{\theta}(B)) \subset sCl_{\theta}(f^{-1}(B))$ .

**(d)⇒(a).** Let  $A$  be any  $\theta$ -semi-open subset of  $X$  and set  $B = Y \setminus f(A) = f(X \setminus A)$ . By (d),  $f^{-1}(sInt_{\theta}(f(X \setminus A))) \subset sCl_{\theta}(f^{-1}(f(X \setminus A))) = sCl_{\theta}(X \setminus A) = X \setminus A$ . Therefore,  $f(X \setminus A) = Y \setminus f(A)$  is  $\theta$ -semi-open and hence  $f(A)$  is  $\theta$ -semi-closed subset of  $Y$ . Thus,  $f$  is contra pre  $\theta$ s-open map.

The proof of the following theorem is similar to the above theorem for the contra pre  $\theta$ s-closed maps.

**Theorem 3.6:** For a map  $f : X \rightarrow Y$ , the following are equivalent:

- a)  $f$  is contra pre  $\theta$ s-closed;
- b)  $f(sCl_{\theta}(A)) \subset (sInt_{\theta} f(A))$  for each subset  $A$  of  $X$ ;
- c)  $sCl_{\theta}(f^{-1}(B)) \subset f^{-1}(sInt_{\theta}(B))$  for each subset  $B$  of  $Y$ ;
- d)  $f^{-1}(sCl_{\theta}(B)) \subset sInt_{\theta}(f^{-1}(B))$  for each subset  $B$  of  $Y$ .

**Theorem 3.7:** Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be a map. Then,

- i) If  $f$  is contra pre  $\theta$ s-open, then  $sCl_{\theta}(f(A)) \subset f(sCl_{\theta}(A))$  for every  $\theta$ -semi-open subset  $A$  of  $X$ .
- ii) If  $f$  is contra pre  $\theta$ s-closed, then  $f(A) \subset sInt_{\theta}(f(sCl_{\theta}(A)))$  for every subset  $A$  of  $X$ .

**Proof: i)** Since  $f$  is contra pre  $\theta$ s-open, then  $sCl_{\theta}(f(A)) = f(A) \subset f(sCl_{\theta}(A))$  for every  $A \in \theta SO(X, \tau)$ .

**ii)** Since  $f$  is contra pre  $\theta$ s-closed and since  $sCl_{\theta}(A)$  is  $\theta$ -semi-closed, then  $f(A) \subset f(sCl_{\theta}(A)) = sInt_{\theta}(f(sCl_{\theta}(A)))$  for every subset  $A$  of  $X$ .

A map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be pre  $\theta$ s-open, if  $f(A)$  is  $\theta$ -semi-open in  $(Y, \sigma)$ , for every  $A \in \theta SO(X, \tau)$ .

Recall, that a map  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called S-closed [4] if  $sCl_{\theta}(f(A)) \subset f(sCl_{\theta}(A))$  for every subset  $A$  of  $X$ .

**Theorem 3.8:** For a map  $f : (X, \tau) \rightarrow (Y, \sigma)$ , the following properties hold,

i)  $f$  is S-closed, whenever  $f$  is contra pre  $\theta$ s-closed and  $sCl_{\theta}(sInt_{\theta}(f(A))) \subset f(A)$  for every  $\theta$ -semi-closed set  $A$  of  $X$ .

ii)  $f$  is pre  $\theta$ s-open, whenever  $f$  is contra pre  $\theta$ s-open and  $f(A) \subset sInt_{\theta}(sCl_{\theta}(f(A)))$  for every  $\theta$ -semi-open set  $A$  of  $X$ .

**Proof: i)** Let  $G$  be a  $\theta$ -semi-closed subset of  $X$ . Since  $sCl_{\theta}(sInt_{\theta}(f(G))) \subset f(G)$  and  $f(G)$  is  $\theta$ -semi-open, then  $sCl_{\theta}(sInt_{\theta}(f(G))) = sCl_{\theta}(f(G)) \subset f(G)$ . So, by [1, Remark 1.2.6],  $f(G)$  is  $\theta$ -semi-closed. Therefore, by [10, Theorem 3.1],  $f$  is S-closed map.

ii) Let  $A$  be a  $\theta$ -semi-open subset of  $X$ . But  $f(A) \subset sInt_{\theta}(sCl_{\theta}(f(A)))$  and  $f(A)$  is  $\theta$ -semi-closed, then  $sInt_{\theta}(sCl_{\theta}(f(A))) = sInt_{\theta}(f(A))$  and hence  $f(A) \subset sInt_{\theta}(f(A))$ . Therefore,  $f(A) = sInt_{\theta}(f(A))$ . So, by [1, Proposition 1.2.2(4)],  $f(A)$  is  $\theta$ -semi-open.

**Lemma 3.3[7]:** If  $Y$  is a regular closed subset of a space  $X$  and  $A \subset Y$ , then  $A$  is  $\theta$ -semi-open in  $X$  if and only if  $A$  is  $\theta$ -semi-open in  $Y$ .

Regarding the restriction  $f|_R$  of a map  $f: (X, \tau) \rightarrow (Y, \sigma)$  to a subset  $R$  of  $X$  we have the following:

**Theorem 3.9:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is contra pre  $\theta$ s-open and  $R$  is a regular closed set of  $(X, \tau)$ , then the map  $f|_R: (R, \tau_R) \rightarrow (Y, \sigma)$  is also contra pre  $\theta$ s-open.

**Proof:** Let  $A$  be a  $\theta$ -semi-open set in the subspace  $R$ . Since  $R$  is regular closed in  $X$ , then by Lemma 3.3,  $A$  is  $\theta$ -semi-open set in  $X$ . Since  $f$  is contra pre  $\theta$ s-open. Therefore,  $f(A)$  is  $\theta$ -semi-closed in  $Y$ . Thus,  $f|_R$  is contra pre  $\theta$ s-open map.

The proof of the following result is not hard, therefore, it is omitted.

**Theorem 3.10:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \gamma)$  be two maps such that  $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ . Then,

- a)  $g \circ f$  is contra pre  $\theta$ s-open, if  $f$  is pre  $\theta$ s-open and  $g$  is contra pre  $\theta$ s-open.
- b)  $g \circ f$  is contra pre  $\theta$ s-open, if  $f$  is contra pre  $\theta$ s-open and  $g$  is S-closed.
- c)  $g \circ f$  is contra pre  $\theta$ s-closed, if  $f$  is S-closed and  $g$  is contra pre  $\theta$ s-closed.
- d)  $g \circ f$  is contra pre  $\theta$ s-closed, if  $f$  is contra pre  $\theta$ s-closed and  $g$  is pre  $\theta$ s-open.

Recall, that a map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is S-continuous [10], if and only if for each  $\theta$ -semi-open subset  $A$  of  $Y$ ,  $f^{-1}(A)$  is  $\theta$ -semi-open in  $X$ .

**Theorem 3.11:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \gamma)$  be two maps such that  $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$  is contra pre  $\theta$ s-closed.

- a) If  $f$  is S-continuous surjection, then  $g$  is contra pre  $\theta$ s-closed.
- b) If  $g$  is S-continuous injection, then  $f$  is contra pre  $\theta$ s-closed.

**Proof:** a) Suppose  $G$  is any arbitrary  $\theta$ -semi-closed set in  $Y$ . Since  $f$  is S-continuous. Therefore, by [10, Theorem 1.1],  $f^{-1}(G)$  is  $\theta$ -semi-closed in  $X$ . Since  $g \circ f$  is contra pre  $\theta$ s-closed and  $f$  is surjective  $(g \circ f)(f^{-1}(G)) = g(G)$  is  $\theta$ -semi-open in  $Z$ . This implies that  $g$  is a contra pre  $\theta$ s-closed map.

b) Suppose  $G$  is any arbitrary  $\theta$ -semi-closed set in  $X$ . Since  $g \circ f$  is contra pre  $\theta$ s-closed,  $(g \circ f)(G)$  is  $\theta$ -semi-open in  $Z$ . Since  $g$  is S-continuous injection,  $g^{-1}((g \circ f)(G)) = f(G)$  is  $\theta$ -semi-open in  $Y$ . This implies that  $f$  is a contra pre  $\theta$ s-closed map.

Arguing as in the proof of Theorem 3.11, we obtain the following result.

**Theorem 3.12:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \gamma)$  be two maps such that  $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$  is contra pre  $\theta$ s-open.

- a) If  $f$  is S-continuous surjection, then  $g$  is contra pre  $\theta$ s-open.
- b) If  $g$  is S-continuous injection, then  $f$  is contra pre  $\theta$ s-open.

**Lemma 3.4[10]:** Let  $(X, \tau)$  be a topological space and  $D$  be a subset of  $X$ . Then  $x \in sCl_{\theta}(D)$  if and only if for every  $\theta$ -semi-open  $A$  of  $x$  such that  $A \cap D \neq \emptyset$ .

**Definition 3.2[5]:** A subset  $D$  of a topological space  $(X, \tau)$  is called  $\theta$ -semi-dense if  $sCl_{\theta}(D) = X$ .

**Theorem 3.13:** For a map  $f: (X, \tau) \rightarrow (Y, \sigma)$ , the following properties hold:

- a) If  $f$  is contra pre  $\theta$ s-open and  $B \subset Y$  has the property that  $B$  is not contained in proper  $\theta$ -semi-open sets, then  $f^{-1}(B)$  is  $\theta$ -semi-dense in  $X$ .

b) If  $f$  is contra pre  $\theta$ s-closed and  $A$  is  $\theta$ -semi-dense subset of  $Y$ , then  $f^{-1}(A)$  is not contained in a proper  $\theta$ -semi-dense set.

**Proof:** a) Let  $x \in X$  and let  $A$  be a  $\theta$ -semi-open subset of  $X$  containing  $x$ . Then  $f(A)$  is  $\theta$ -semi-closed and  $Y \setminus f(A)$  is a proper  $\theta$ -semi-open subset of  $Y$ . Thus,  $B \not\subset Y \setminus f(A)$  and hence there exists  $y \in B$  such that  $y \in f(A)$ . Let  $z \in A$  for which  $y = f(z)$ . Then  $z \in A \cap f^{-1}(B)$ . Hence  $A \cap f^{-1}(B) \neq \emptyset$  and thus by Lemma 3.4,  $x \in sCl_{\theta}(f^{-1}(B))$ . Hence  $f^{-1}(B)$  is  $\theta$ -semi-dense in  $X$ .

b) Assume that  $f^{-1}(A) \subset O$  where  $O$  is a proper  $\theta$ -semi-open subset of  $X$ . Then, we have that  $f(X \setminus O)$  is a non-empty  $\theta$ -semi-open set such that  $f(X \setminus O) \cap A = \emptyset$ , which contradicts the fact that  $A$  is  $\theta$ -semi-dense.

**Lemma 3.5[6]:** Let  $X_1$  and  $X_2$  be two topological spaces and  $X = X_1 \times X_2$ . Let  $A_i \in \theta SO(X_i)$  for  $i = 1, 2$ , then  $A_1 \times A_2 \in \theta SO(X_1 \times X_2)$ .

**Definition 3.3[7]:** A space  $X$  is said to be strongly semi- $T_2$  if and only if for each two distinct points  $x$  and  $y$  in  $X$ , there exists two disjoint  $\theta$ -semi-open sets  $A$  and  $B$  in  $X$  containing  $x$  and  $y$ , respectively.

**Theorem 3.14:** If  $X$  is a strongly semi- $T_2$  space and  $f : X \rightarrow Y$  is contra pre  $\theta$ s-open map, then the set  $A = \{(x_1, x_2) : f(x_1) = f(x_2)\}$  is  $\theta$ -semi-closed in the product space  $X \times X$ .

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