VALIDATION OF THE TOTAL RESISTANCE HEAT DISSIPATION MODEL FOR HEAT TRANSMISSION THROUGH ANNULAR FINS

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Abstract

An experimental investigation of the heat transfer in annular fins of constant thickness was carried out to prove the total resistance model suggested by Kahwaji [4] , given in equation (1) in the introduction . The experiments covered both the natural convection and forced convection heat transfer modes using fins of different materials and dimensions. Different Ra and Re numbers also achieved through varying the power input to the fins and the speed of the air flowing through the fin assembly. The results indicated good agreement between the suggested model and the experimental findings. Calculated and measured heat flux was found to be less than (8.33%) in the natural convection tests and (11%) in the forced convection tests. The maximum experimental error was estimated at about (6.33%). A numerical solution, based on the Gauss-siedel technique, was also derived and used to support the results.

Key words: Heat transfer, annular fin, natural and forced convection, heat transfer coefficient

تحقيق نموذج المقاومة الكلية لانتقال الحرارة في الزعانف الحلقية عملياً

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الخلاصة

تم في هذا البحث أجراء دراسة عملية لانتقال الحرارة خلال مجموعة من الزعانف الحلقية ذوات السمك الثابت لاثبات صحة المقاومة الكلية لانتقال الحرارة من خلال المقارنة بمعادلة مقترحة من قبل الباحث Kahwaji [4] تمثل نموذج المقاومة الكلية والمعطاة في المعادلة (١) من المقدمة.

استخدمت طريقتين في اجراء التجارب وهما الحمل الطبيعي والحمل القسري باستخدام معدنين مختلفين بالموصلية الحرارية والسمك وهما النحاس والبراص ، إذ باستخدام الحمل الطبيعي تم الاعتماد على تغيير اطوال الزعنفة والفيض الحراري الداخل الى الانبوبة المزعنفة فيما تم الاعتماد على تغيير سرعة الهواء المسلطة والفيض الحراري الداخل باستخدام الحمل القسري كمتغيرات لتحقيق العمل . وبعد اجراء التجارب تم التوصل الى تحقيق التطابق بين الحلين العملي والنظري ولكلا الطريقتين وبالتالي إثبات صحة نموذج المقاومة الكلية لانتقال الحرارة الموضحة في المعادلة أعلاه. وقد بلغت اكبر نسبة خطأ بين الحلين (٨.٣٣%) في تجارب الحمل الطبيعي و(١١%) في تجارب الحمل القسري ، وبلغت أكبر نسبة للخطأ نتيجة أجهزة القياس المستخدمة ولكلا الطريقتين (٦.٣٣%) ، وكمقارنة إضافية تم استخدام الطريقة العددية لانتقال الحرارة خلال الاتجاهين القطري والمستعرض بطريقة الفروق المحددة وباستخدام تقنية كاوس سيدل اذ ان هذه الطريقة اثبتت صحة النتائج.

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Introduction

It has become a standard procedure to use extended surfaces in cases where the designer is faced with the problem of high convective resistance on one or both sides of a prime heat exchange surface, such extended surfaces are termed fins. A fin can be defined as an extra surface added to a prime surface to increase the heat transfer per unit of its prime surface area.

Researchers were mainly concerned with the calculation of heat transfer from different types of fins and have used many assumptions to simplify the calculation process. Three types of solution methods were explored, analytical, numerical and experimental.

Analytical solutions were carried out with vary levels of simplifying assumptions yielding heat dissipation correlation's that

various in their complexity from simple ones to more complex ones. Numerical solutions mainly used to fill gaps in experimental results or to investigate the relief of certain simplifying assumptions.

Higges [1] studied the heat transfer from annular fins of triangular profile with variable heat transfer coefficient from the base to the tip, he found that the increase in the heat transfer coefficient will causes the decrease in the efficiency of the fin. Irey [2] investigated circular fins and reported that the one - dimensional approximation is only valid for small Biot number (Bi=hr/k).

Keller and Somers [3] presented an analytical solution for annular fins with two dimensional heat flow, however, their choice of parameters led to the conclusion that the approximation is valid for length to thickness ratio greater or equal to ten.

Kahwaji [4] conducted a numerical and electrical analogue study of the thermal performance of annular fins of constant thickness under the one and two dimensional heat flow assumptions. He suggested a new simple method for correlating the fin rate of heat transfer which depends on the grouping of the different thermal resistance of the fin in a (driving force / resistance) form model. The suggested resistance is given as:

$$TR_{th} = \left[\left(\frac{\ln(r_o/r_i)}{2\pi kW}\right)^2 + \left(\frac{W}{4\pi k(r_o^2 - r_i^2)}\right)^2\right]^{\frac{1}{2}} + \frac{1}{2\pi h(r_o^2 - r_i^2)} \quad \ldots(1)$$

Where TR_{th} is the total resistance of the fin and the term under the sequare root is the two - dimensional material resistance while the other term is the surface resistance, as shown in figure (1).

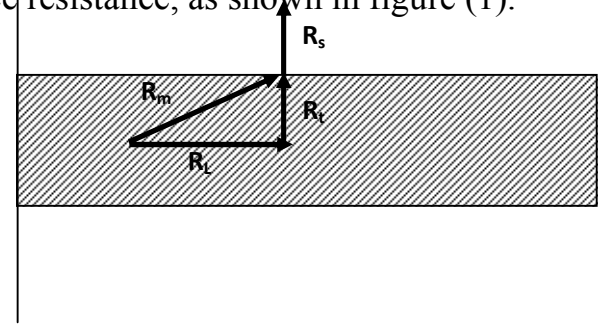


Figure (1) The resistance components for an annular fin of constant thickness

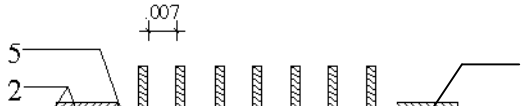
The above form of correlation was found to give an accurate representation of the heat transfer from the fin. Moreover, the assumption of one - dimensional heat flow through the fins was found to be valid when the total resistance (TR_{th}) is greater than 835°C/kW.

Surveying the literature indicated that the above model given by Kahwaji for the calculation of heat dissipation from fins is the simplest to use compared to those given by [2] and [3] where either charts or a computer should be used in the heat rate estimation.

In this paper, an experimental investigation is carried out to test the validity of the above model under both natural and forced convection heat transfer conditions from annular fins of constant thickness. The work includes conducting experiments to measure the heat dissipation from the fin as well as the convective heat transfer coefficient on the fin surface.

Experimental apparatus:

The heart of the experimental rig is the finned tube. The design concept is illustrated in the longitudinal sectional view presented in figure (2). The finned tube synthesized by assembling spacer rings between circular fins. Spacer rings were machined from a solid rod to a chive an isothermal surface condition at the fin root and the surface of the



cylinder. The fins and rings were polished in order to reduce the surface emissivity and thus to minimize the heat losses by radiation. The assembly of the apparatus was performed with the successive rings and fins slipped over precisely machined manderllike steel tube (i.e the assembly tube) and with pressure applied to the stack to insure perfect contact. Manufactured heater core (glass tube) inserted concentrically inside the assembly tube was filled with sand particles to provide both circumferential and axial heating uniformity. The ends of the element were insulated using cork pieces. The experiments covered the natural convection and forced convection heat transfer modes using seven fins and eight rings with different materials and dimensions.

Different materials and dimensions are used to insure a wide range of thermal conditions as shown in tables (1) and (2).

Table (1) The different parameters used in the natural convection tests

Natural Convection			
Material	Thickness (mm)	Radius (mm)	Heat flux (W)
Copper	2	20-25-30-35	10-50
Brass	1	20-25-30-35	10-50

Table (2) The different parameters used in the forced convection tests

Forced Convection				
Material	Thickness (mm)	Radius (mm)	Heat flux (W)	Velocity (m/s)
Copper	2	35	20-145	4-8.5
Brass	0.5	35	20-132	4-8.5

The input power is varied to cover a Rayleigh number range of ($1\text{-}5\text{x}10^6$). The finned tube was equipped with eighteen (0.2 mm copper-constantan) thermocouples. Sixteen of them are distributed on the surface

of four fins, each fin has four thermocouples arranged radially making an angle of (90°) from those on other fins to cover the four basic directions. Two more thermocouples are used to measure the base temperature on the rings.

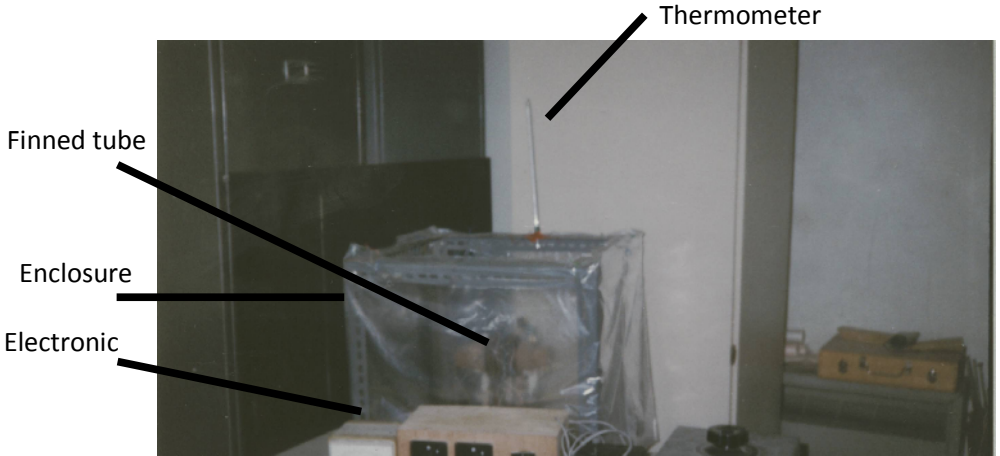
Figure (3) shows the natural convection apparatus, while figure (4) shows the forced convection apparatus. The forced convection tests covered a Reynolds number range of (19-43x10^3) based on the average path length of the flow along the fin and an air speed range between 4 - 8.5 m/s.

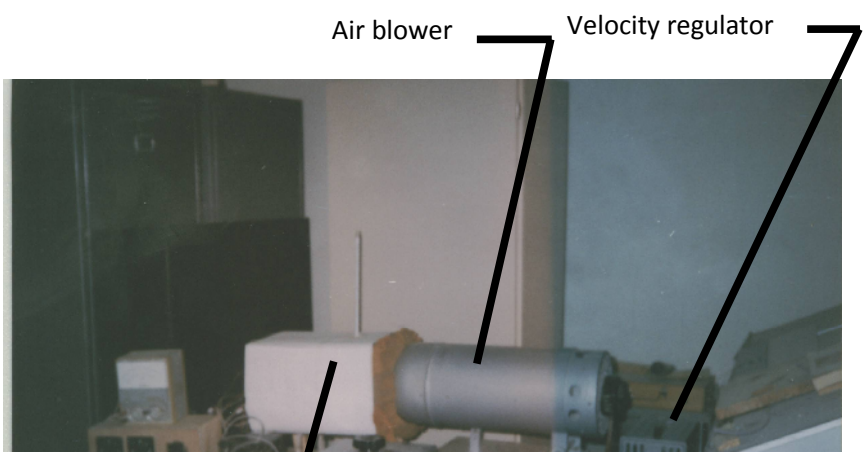
The end loss from the assembly is estimated by insulating the circumference with a thick layer of glass wool and measuring the power dissipation at different levels of temperature difference between the assembly and environment, i.e:

$$Q_e = Q_s - Q_c \qquad \ldots(2)$$

Where Q_e is the end loss, Q_s is the supplied heat and Q_c is the circumferential heat transfer given by well-known formula:

$$Q_c = \frac{2\pi k_g L_g (T_b - T_f)}{\ln(r_{go} / r_{gi})} \qquad \ldots(3)$$




Air blower
Velocity regulator

Finally, a Kline - McLintock [5] type uncertainty analysis is applied to reveal that the uncertainty in Nu is within (6.3%) for the range of variables covered in the experiment. Which is considered very good for such experimental work.

Method of Calculation

The heat dissipated by the fin is estimated from the experimental measurement as:

$$Q_{exp} = (Q_s - Q_e - nQ_b) / m \quad \ldots(4)$$

From which the experimental total resistance is estimated as:-

$$TR_{exp} = \frac{1}{(Q_{exp}/(T_b - T_f))} \quad \ldots(5)$$

Next the fin efficiency and heat transfer coefficients are calculated iteratively from:

$$h = Q_{net}/((T_b - T_f)(nA_b - mA_f\eta)) \quad \ldots(6)$$

Where,

$$\eta = \frac{2r_i}{m(r_o^2 - r_i^2)}\left(\frac{I_1(mr_o)K_1(mr_i) - K(mr_o)I_1(mr_i)}{I_o(mr_i)K(mr_o) + I_1(mr_o)K_o(mr_i)}\right) \quad \ldots(7)$$

From which $T.R._{th}$ is calculate from equation (1) and Q_{th} is calculated as :

$$Q_{th} = \left(\frac{T_b - T_f}{TR_{th}} \right)$$

…(8)

Results and Discussion

Convection Heat Transfer Coefficient

Figure (5) shows the Nu-Ra relation ship for the natural convection tests. Where Ra is based on the average path length for the flow along the fin as the scale length.

The data may be best represented by :

$$Nu = 0.451(Ra)^{0.25}$$

…(9)

Where : $Nu = \frac{h.D_{av}}{k_f}$ and $Ra = \frac{\beta g(T_b - T_f)D_{av}{}^3.Pr}{\nu^2}$

This correlation is found to be in line with the vertical flat plate results given by [6] as:

$$Nu = 0.59(Ra)^{0.25}$$

…(10)

The reduced constant, hence Nu, in the present case is due to the presence of the tube which retards the flow near the center and the interaction of the two boundary layers on the opposite face to face fins.

The results of the forced convection heat transfer tests are shown in figure (6). Analyzing the data indicated that Nu may be best represented, in this case, by the following correlation:-

$$Nu = 0.0695(Ra)^{0.182}(Re)^{0.644}(Bi)^{0.363} \qquad \ldots(11)$$

Where :

$$Re = \frac{V.D_{av}}{\nu} \quad , \quad Bi = \frac{h.D_{av}}{k_m}$$

In this correlation, Rayliegh number describes the driving force caused by the temperature difference between the base and the surrounding air (boundary), Reynolds number describes the air velocity effects and Biot number takes into account the effect of the fin efficiency, i.e the effect of material conductivity on the temperature distribution along the fin. From the above correlation, it is obvious, that low conductivity fins should show lower efficiencies, higher decay in longitudinal temperature distributions and hence a higher averaged (h) for a given heat dissipation.

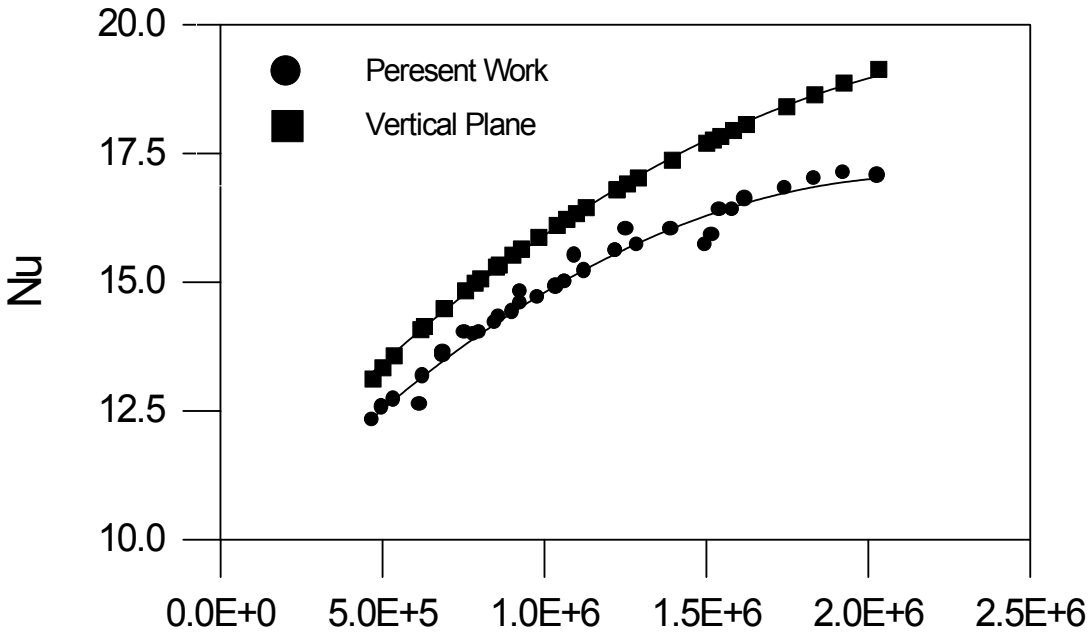
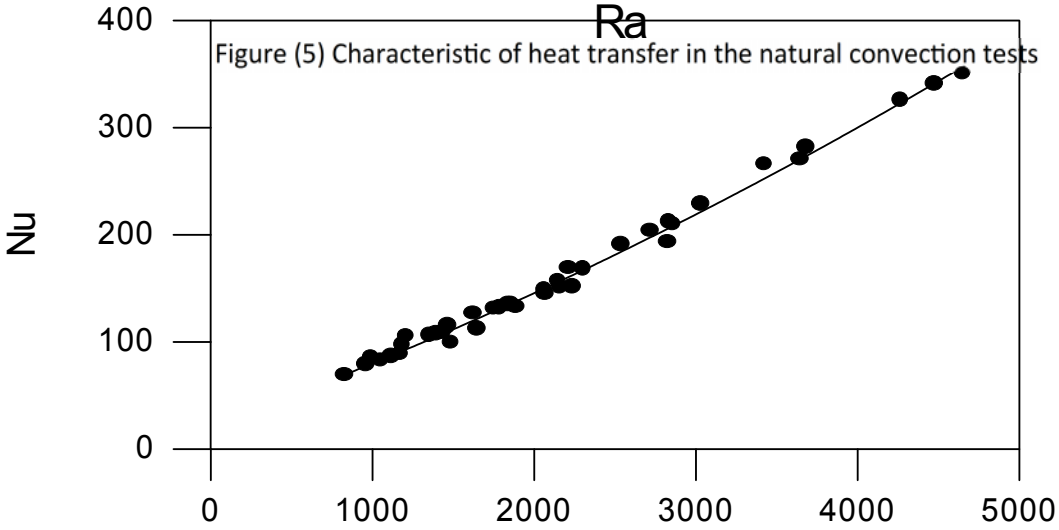


Figure (5) Characteristic of heat transfer in the natural convection tests



$$(Ra)^{0.182}(Re)^{0.644}(Bi)^{0.363}$$

Figure (6) Characteristic of heat transfer in the forced convection tests

Validation of the Total Resistance

Figure (7) shows a comparison of the fin heat dissipation in the natural convection mode as obtained from experimental procedure, the total resistance model and a two dimensional numerical model built for this purpose. The vertical axis represents the heat transfer per degree temperature difference between the fin base and ambient air.

The three solutions fell within an envelope of (8.3%) with the numerical solution results falling between the experimental results on top and the total resistance model results at the lower ends. If the calculated (6.3%) uncertainty in the experimental results is taken into consideration as well as the insulated tip assumption in the theoretical calculations, the agreement can be considered very good for all practical purposes.

Figure (7) shows also the suggested equation of the total resistance model, i.e.: $Q_f = \frac{(T_b - T_f)}{TR}$

…(12)

Which indicates the adequacy of the model as well as its simplicity. Compared to the first total resistance model of Kahwaji [4], given by:

$$Q_f = 1.2218(TR_{th})^{-1.018}(T_b - T_f)$$

…(13)

the agreement is good where the difference can be attributed to the insulated tip assumption used by reference [4].

Figure (8), shows a comparison of the fin heat dissipation in the forced convection mode as obtained from the experimental tests as in figure (7), the total resistance model and two dimensional numerical model are plotted in this figure for this purpose. The three solutions fell with an envelope of (11%). If the calculated (6.3%) uncertainty in the experimental results is taken into consideration as well as the insulated tip assumption in the theoretical calculations, the agreement can be considered to be good for all practical purposes.

Figure (9) shows the combined natural and forced convection heat transfer results plotted together. All points fell on the same straight line which indicates the sufficiency of the proposed model and it's adequacy to describe the phenomena of heat transfer through circular fins. The agreement between the suggested model, the experimental results and the numerical model is also in good agreement. Extending the numerical model results and suggested model results beyond the experiment range

shows good agreement between them as shown in this figure. This further substantiates the validity of the suggested model.

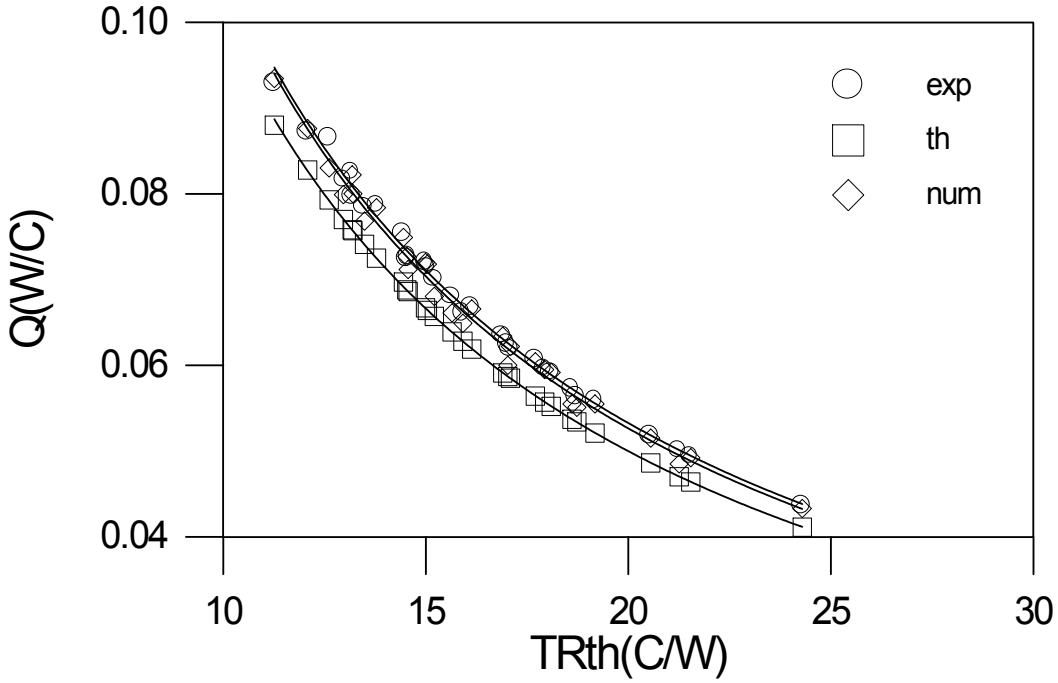


Figure (7) The variation the total resistance with heat transfer in the natural convection tests

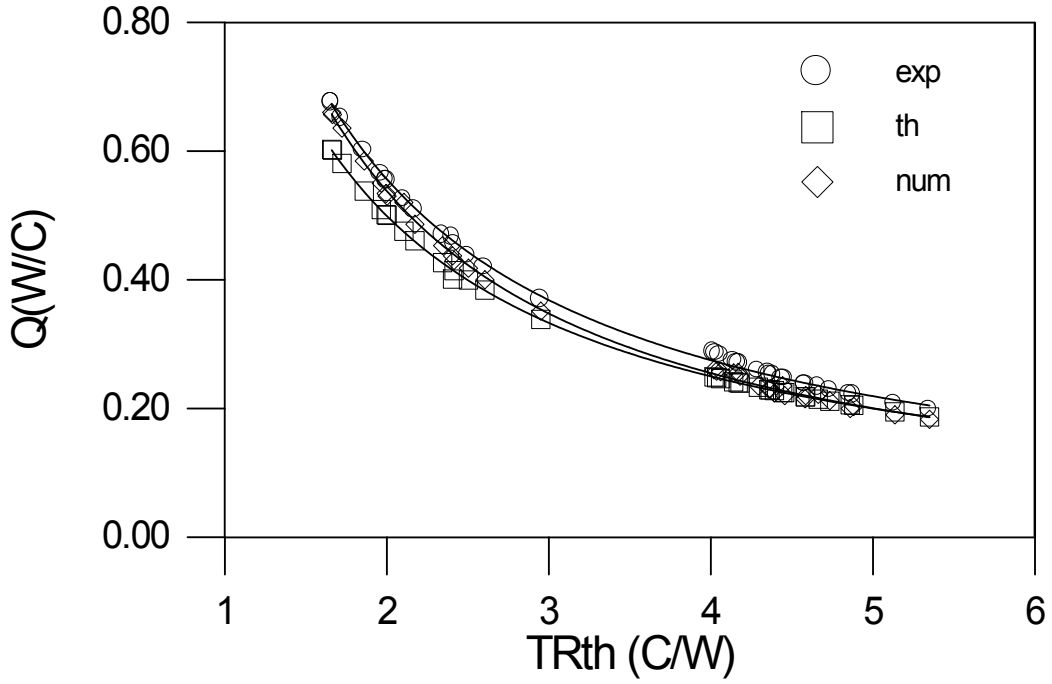


Figure (8) The variation the total resistance with heat transfer in the forced convection tests

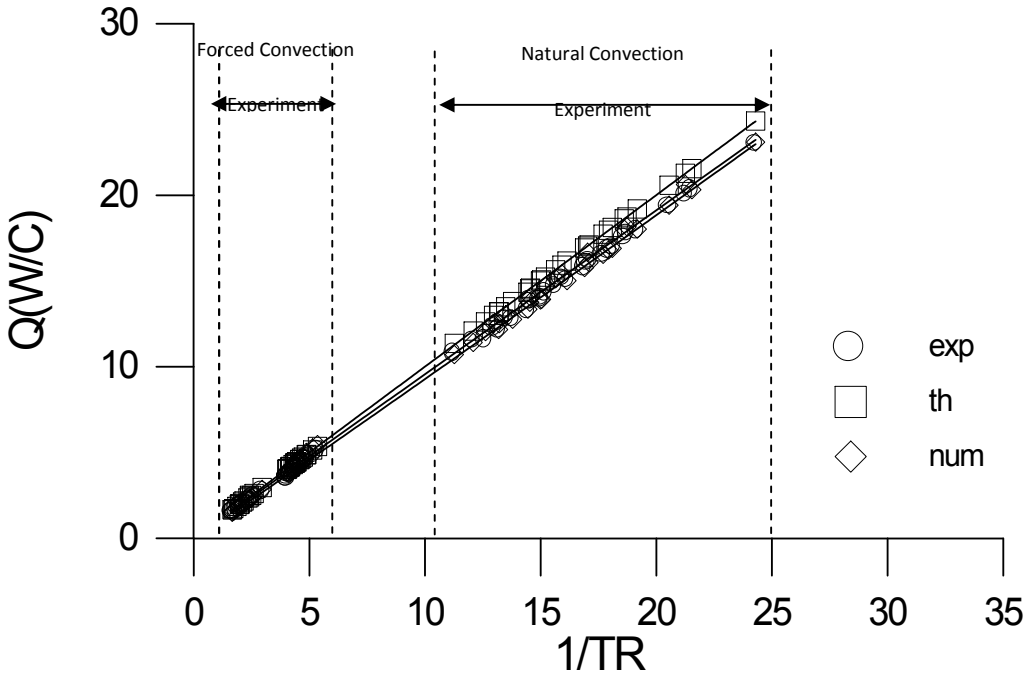


Figure (9) The linear variation of the heat dissipation with the total resistance in the forced and natural convection tests

Conclusions:

1- The agreement may be considered acceptable between the experimental results and the heat dissipation model suggested by Kahwaji [4] ie:

$$Q = (Tb - Tf) * TR\text{-}1$$

The error was found to be about (8.3%) in the natural convection tests and about (11.0%) in the forced convection tests. Hence the model may be considered useful in fins heat transfer analysis due to its simplicity and accuracy.

2- In the natural convection tests, Nu was found as a function of Ra with the following correlation obtained:

$$Nu = 0.451(Ra)^{0.25}$$

While it was found as a function of Ra, Re and Bi numbers in the forced convection tests as follows:

$$Nu = 0.0695(Ra)^{0.182}(Re)^{0.644}(Bi)^{0.363}$$

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Nomenclature

A_b : Surface area of the rings (m^2)

A_f : Surface area of fin (m^2)

Bi : Biot number

D_{av} : Average diameter

g : (m/s^2)

h : Heat transfer coefficient ($W/m^2.°C$)

I_o,I_1,K_o,K_1 : First and second mode of Bessel function

k : Thermal conductivity (W/m.°C)

k_f : Thermal conductivity of fluid (W/m.°C)

k_g : Thermal conductivity of glass wool (W/m.°C)

k_m : Thermal conductivity of material (W/m.°C)

L_g : Length of glass wool (m)

m : Number of rings

n : Number of fins

Nu : Nusselt number

Q_b : The heat transfer from rings experimentally (W)

Q_c : The heat losses from the two ends (W)

Q_e : Heat transfer from glass wool (W)

Q_{exp} : The heat transfer from fin experimentally (W)

Q_f : The heat transfer from fin (W)

Q_{net} : the heat transfer from the finned tube with out losses (W)

Q_s : Input heat flux (W)

Q_{th} : The heat transfer from fin theoretically (W)

Ra : Rayliegh number

Re : Reynolds number

R_L : The longitudinal resistance of fin (°C/W)

R_m : the material resistance of fin (°C/W)

R_t : The tangential resistance of fin (°C/W)

R_S : The surface resistance of fin (°C/W)

r_i : Inner radius of fin (m)

r_o : Outer radius of fin (m)

r_{gi} : Inner radius of glass wool (m)

r_{go} : Outer radius of glass wool (m)

Pr : Prantil number

T_b : base temperature (°C)

T_f : Fluid temperature (°C)

TR : Total resistance (°C/W)

TR_{exp} : Total resistance experimentally (°C/W)

TR_{th} : Total resistance theoretically (°C/W)

W : Thickness of fin (m)

V : Air velocity (m/s)

η : Fin efficiency

ν : Kinematics viscosity (m^2/s)

β : (1/°C)

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